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CONTENTS

Chemistry. — *On reversible phase-transitions in solid refractory metals.* By Prof. Dr. F. M. JAEGER.

(Communicated at the meeting of June 29, 1940.)

§ 1. During the last twenty years a great number of systematical investigations have been made in the Laboratory of Inorganic and Physical Chemistry of the Groningen University, concerning the phase-transitions occurring in refractory metals at temperatures between 0° C. and about 1650° C., whilst the accompanying phenomena were studied in detail. The metals here considered all belong to the 4th till 8th group of the periodic table: more especially they bear the atomic numbers: 22 to 28, 40 to 46

4th Group	5th Group	6th Group	7th Group	8th Group				
22	23	24	25	Atomic Number	26	27	28	IV th per.
Ti	*V*	*Cr*	*Mn*	Element	*Fe*	*Co*	*Ni*	
47.9 (5 isot.)	50.95 (1 isot.)	52.01 (4 isot.)	54.93 (1 isot.)	Atomic Weight	55.84 (4 isot.)	58.94 (2 isot.)	58.69 (5 isot.)	
1730°C.	1715°C.	1765°C.	ca. 1245°C.	Meltingpoint	1530°C.	1490°C.	1455°C.	
2 (3)	1	2 (3)	3	Number of Solid States	4 (2)	3 (2)	3 (2)	
40	41	42	43	Atomic Number	44	45	46	Vth per.
Zr	*Nb*	*Mo*	*Ma*	Element	*Ru*	*Rh*	*Pd*	
91.2 (5 isot.)	92.91 (1 isot.)	95.95 (7 isot.)	(ca. 99) (—)	Atomic Weight	101.7 (6 isot.)	102.91 (2 isot.)	106.7 (6 isot.)	
2130°C.	1950°C.	2625°C.	—	Meltingpoint	2450°C.	1966°C.	1554.5° C.	
2 (3)	1	1	—	Number of Solid States	4	2	1	
72	73	74	75	Atomic Number	76	77	78	VIth per.
Hf	*Ta*	*W*	*Re*	Element	*Os*	*Ir*	*Pt*	
178.4 (5 isot.)	180.88 (1 isot.)	183.92 (4 isot.)	186.31 (2 isot.)	Atomic Weight	191.5 (6 isot.)	193.1 (2 isot.)	195.23 (5 isot.)	
2230°C.	3030°C.	3370°C.	3440°C.	Meltingpoint	2700°C.	2454°C.	1773.5° C.	
2	1	1	1	Number of Solid States	1	1	1	

and 72 to 78, including the three series of elements ranging from *titanium* to *nickel*, from *zirconium* to *palladium* and from *hafnium* to *platinum*. Their meltingpoints (under the pressure of 1 atm.) all lie between about 1500° and 3500° C.; many of them belong to the technically most important materials, either in the pure or in the alloyed state.

After the recently [1]) finished investigation of *cobaltum* in this respect, the work which had to be done concerning the 8th group can now be considered as accomplished; so that, therefore, a summarizing review of the results obtained in the latter group, seems not to be out of place here.

§ 2. A few additional remarks may be made here about the methods applied in these investigations. As the purpose was to collect not only the necessary data about the accurate situation of the characteristic transition-temperatures, but also to make sure that no phase-transitions in the solid state could be overlooked, — it appeared from the beginning adviseable to determine those data by *several different* experimental methods in such a way, that the values obtained could be controlled by mutual comparison; moreover, that these values should preferentially not be gained by *separate* observations at *arbitrarily selected* temperatures, but by an experimental device which would allow us to acquire a record of the dependence of the studied properties on *all intermediate* temperatures. For this reason the method of the optically-coupled double-galvanometer, as first designed by SALADIN and LE CHATELIER, in combination with a photographic registration of the curves obtained, was modified in several directions; so that now it allows the direct record on the photographic plate of the curves for the *relative heat-capacity*, the *thermal expansion*, the *electric resistance* and the *thermo-electrical* behaviour of the metals studied in function of the temperature (as abscissa). As soon as in this way the true situation of the remarkable points on the curves is sufficiently ascertained, the accurate measurements of the absolute specific heats by means of the metalblock-calorimeter at carefully selected intervals of the temperature are made in the way that has already often been described [2]). In these experiments

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[1]) F. M. JAEGER, E. ROSENBOHM and A. J. ZUITHOFF, Rec. d. trav. d. Chim. d. Pays-Bas, **59**, 831 (1940); conf. also: F. M. JAEGER and A. J. ZUITHOFF, these Proceed., **43**, 815 (1940).

[2]) For these methods, compare e.g.: F. M. JAEGER, E. ROSENBOHM and J. A. BOTTEMA, Rec. Trav. chim. d. Pays-Bas, **52**, 61 (1933); F. M. JAEGER and W. A. VEENSTRA, ibid., **53**, 677 (1934); F. M. JAEGER, E. ROSENBOHM and R. FONTEYNE, ibid., **55**, 615 (1936); E. ROSENBOHM, Physica, **5**, 385 (1938); F. M. JAEGER, E. ROSENBOHM and J. A. BOTTEMA, Rec. d. Trav. d. Chim. d. Pays-Bas, **57**, 1137 (1938); F. M. JAEGER and E. ROSENBOHM, Rec. trav. chim., **47**, 513 (1928); id. Physica **6**, 1123 (1939); F. M. JAEGER and E. ROSENBOHM, Rec. trav. chim. **51**, 1 (1932); F. M. JAEGER and W. A. VEENSTRA, Proc. Kon. Akad. v. Wetensch., Amsterdam, **37**, 280 (1934); M. EWERT, Proc. Kon. Akad. v. Wetensch., Amsterdam, **39**, 833 (1936); F. M. JAEGER, E. ROSENBOHM and A. J. ZUITHOFF, Rec. trav. chim. **57**, 1313 (1938); A. J. ZUITHOFF, Rec. trav. chim. **58**, 131 (1939); etc.

the metals were always studied either enclosed in the usual evacuated platinum crucibles of special construction, or within an electrically heated high-vacuum furnace; the metals must beforehand be freed as completely as possible from adherent or absorbed gases.

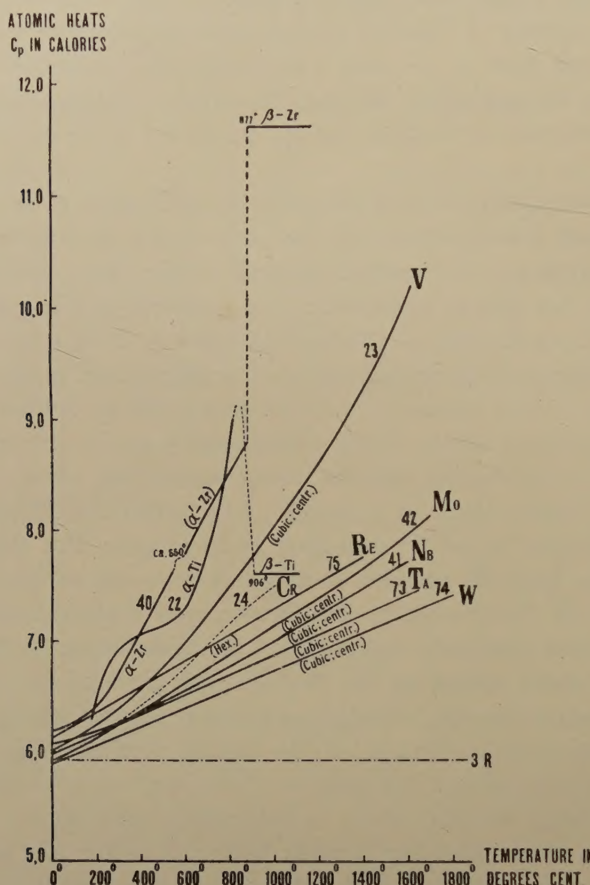

Fig. 1. *The atomic heats C_p of some elements of the 4th, 5th, 6th and 7th group.*

§ 3. With the exception of some elements of groups IV to VII, like *chromium, manganese, masurium* and *hafnium,* — which were not available either in sufficient quantities or in the necessary degree of purity, — the numerical data for all the other metals were finally obtained. It can here be remarked, that the metals of the 5th, 6th and 7th groups, in general proved *not* to occur in more than a *single* crystalline state; thus e.g. in the case of *vanadium, niobium, tantalum, molybdenum, tungsten, rhenium,* etc. (Fig. 1). This fact is contrary to the behaviour of the metals of the 4th group, which all very conspicuously manifest the phenomenon of multiple states. But *all* these elements, — just as is the case with most other metals, — prove to have values for their atomic heats C_p, which, — even at the

lower temperatures (50°—100° C.), — are far superior to the theoretical limiting value of $3R$ (= 5,965) calories. —

§ 4. If the elements of the 8th group now are considered here more in detail, — then the general aspect is quite a different one. In the first place it can be stated, that here only the *last tetrade* of elements of this group: *platinum, iridium, osmium* and *palladium, do not manifest the phenomenon of multiple states.* Indeed, they show *no trace* whatsoever of a change of their internal condition between 0° and 1650° C.; they preserve their original structure even at the highest temperatures considered, — as may be seen (Fig. 2) from the perfectly smooth and uninterrupted shape of all curves representing the numerical values of their physical properties in their dependence on the temperature, e.g. of their specific heats, electrical resistance, etc. Especially in the case of *platinum,* — the mean specific heats of which are now known with an accuracy of 0,1 % of their values, — this fact is of the utmost practical importance, because this indispensable metal is used in *all* high-temperature investigations as a *standard*-material for all comparative and absolute measurements; and its different physical properties in function of the temperature must, therefore, everywhere be

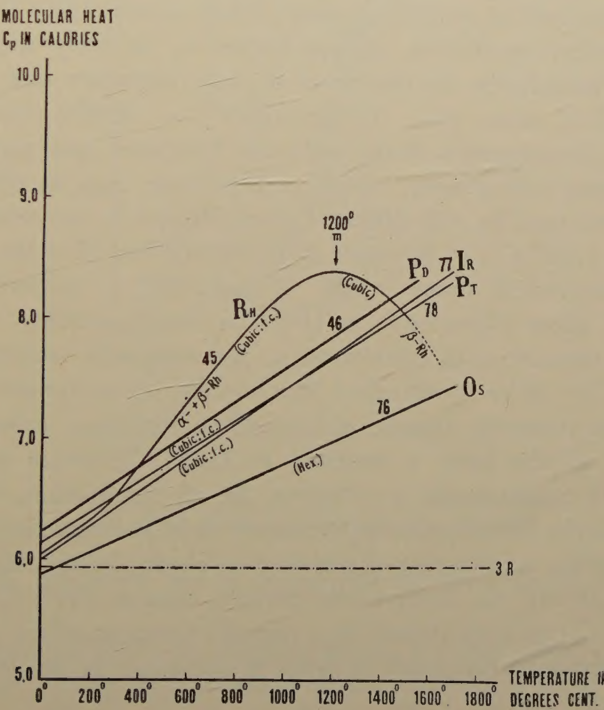

Fig. 2. *The atomic heats C_p of five elements of the 8th group.*

taken into account. The certitude, that up to the highest temperatures below the meltingpoint all physical properties of *platinum* remain *continuously*

variable with the temperature, is, therefore, of essential interest to the practice of high-temperature experiments.

An apparent, small discrepancy in the c_p-t-curve of palladium, — originally found as a weak maximum — was afterwards proved as *non-existent*, — it only being caused by the adoption of an erroneous figure for the meltingpoint of the metal by DAY and SOSMAN in their work on the nitrogengasthermometer: a redetermination of the meltingpoint of *palladium* in this laboratory yielded the value: 1554°,5 C. instead of the old value: 1549°,2 C. After the corresponding corrections for the indications of our thermocouples had been applied, the deviations of the c_p-t-curve of *palladium* proved to have completely disappeared: *palladium*, up to its meltingpoint, therefore, exhibits a *quite normal* behaviour, — just like the three other platinummetals here considered.

§ 5. In rigorous contradistinction now to the behaviour of the four *latter* elements of the 8th group, is that of the *first tetrade*: *iron*, *cobaltum*, *nickel* and *ruthenium*. For these metals all show the phenomenon of multiple phase-transitions in the solid state in a most conspicuous way. Thus *iron* shows four, *cobaltum* three, *nickel* two (or three) and *ruthenium* no less than four, perhaps even five, of such successive reversible changes in the solid state; the field of stability of each of them is, according to the rules of thermodynamical equilibrium, always limited by as many sharply defined temperature-boundaries. In the cases of *iron*, *cobaltum* and *nickel* there are, in the first place, their "CURIE-points", — i.e. the temperatures at which their *ferromagnetic states* suddenly disappear and make place for weakly-paramagnetic phases; which, in their turn, may at other temperatures be substituted by still different ones. Whilst in the case of *iron* the CURIE-point (760° C.) is, however, *first* reached and *then* the other transformations afterwards occur at 906° C. and 1401° C. successively, — the sequence of these phenomena in *cobaltum* (and probably in *nickel*) is exactly *the reverse*: with *cobaltum* e.g. *ferromagnetic α-cobaltum* proves at 445° C. *first* to be transformed into equally *ferromagnetic β-cobaltum*, with an accompanying change of crystalline structure; subsequently the CURIE-point of the latter is reached at 1125° C., where *β-cobaltum* is changed into paramagnetic *γ-cobaltum*. In all three cases, however, *the transitions at the* CURIE-*point appear never to be accompanied by a change of the crystalline architecture* of the original phase. In the case of *cobaltum* the change at 445° C. (with *nickel* perhaps also at 345°; CURIE-point at 355°—360° C.) is accompanied by a typical hysteresis-effect, — the details of which have been discussed in detail in our paper in the *Recueil* [1]).

Most remarkable with respect to its phase-transitions is *ruthenium*, —

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[1]) F. M. JAEGER, E. ROSENBOHM and A. J. ZUITHOFF, Rec. d. trav. d. Chim. d. Pays-Bas, **59**, 831 (1940); conf. also: F. M. JAEGER and A. J. ZUITHOFF, Proc. Ned. Akad. v. Wetensch., Amsterdam, **33**, 815 (1940).

which in its general points appears to behave in a similar way as *iron*. But *ruthenium* is *paramagnetic* and, therefore, does *not* possess a CURIE-temperature. The metal has three or four transition-points at 310°, 1035°, 1200° and 1500° C. respectively; the nature of the transition-point at 310° C., deduced from the recent measurements of the temperature-coefficient of the electrical resistance, still remains problematic [1]). The corre-

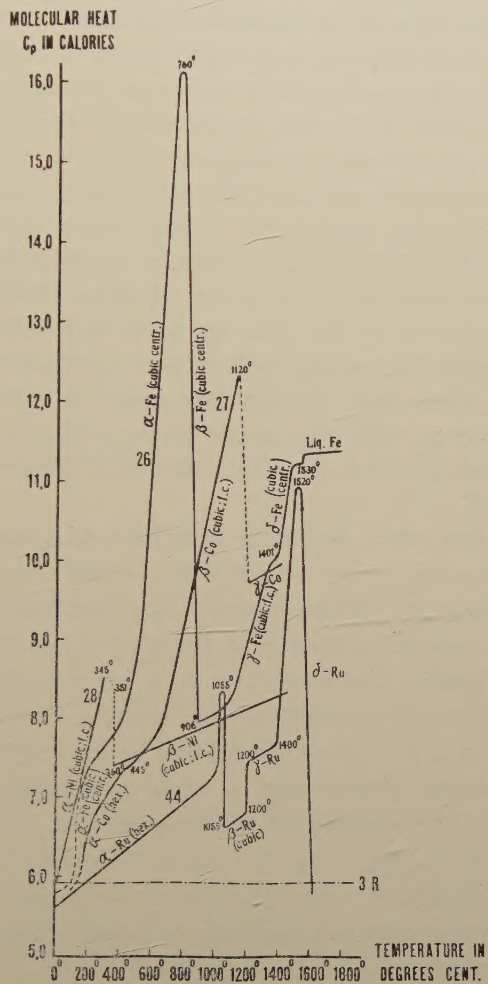

Fig. 3. *The atomic heat of the four first elements of the 8th group.*

sponding transformations at these different temperatures, — as well on heating as on cooling, — all occur *rather rapidly*, without retardation-phenomena of any appreciable significance. Recently this fact was once more qualitatively corroborated by us during a study of the electrical

1) There seems *not* to occur a change of the crystalline structure of the metal at this temperature.

resistance of the metal in its dependence on the temperature, which will be published later.

§ 6. Thus it becomes evident, that the *first* tetrade of metals of the 8th group manifests the phenomenon of enantiotropous phase-transitions to a high degree, whilst the *latter* tetrade of this group, does *not* show it at all. Between these two tetrades, however, the element *rhodium* is placed, — like a central link between the two groups. Now *rhodium* shows a c_p-*t*-curve of an entirely different character, as it consists of *a single, steeply rising, smooth curve* with an unmistakable *maximum* at about 1204° C.; then subsequently the curve at higher temperatures falls-off rather rapidly. As a consequence of a number of experimental facts, earlier investigators have already come to the conclusion, that the internal condition of *rhodium gradually* changes to an appreciable degree, when the metal is heated between 1200° and 1300° C. Thus already in 1908 MENDENHALL and INGERSOLL drew attention to the fact, that at about 1200° C. an abnormal increase in the radiation of the glowing metal is observed; and in 1931 DIXON stated a similar behaviour with respect to the photo-electrical and the thermo-ionic phenomena. In this laboratory an analogous fact was observed in connection with the temperature-coefficient of the electrical resistance, measured in a high vacuum, which also has a maximum between 1200° and 1300° C.

Although the fact, that the inner condition of *rhodium changes* on heating can hardly any longer be doubted, — this change appears to be a *gradual* one. There is nowhere observed any discontinuity in the curves representing the measured properties in function of the temperature. *X*-ray examinations of heated *rhodium*-wires in a vacuum, however, seem to indicate with much more probability, that metallic *rhodium* at each temperature consists of *a homogeneous mixture* of *two different cubic forms* of the metal in variable proportions, — their ratio being characteristic for each temperature. The quantity of the face-centred, cubic α-form of the metal predominates at the *lower*, the cubic β-form (with a simple cubic elementary cell), on the contrary, prevails at the *higher* temperatures. On the other hand it proves not to be possible to isolate either of the two forms in a completely pure state. At each temperature the metal is a homogeneous mixture of α-*Rh* + β-*Rh*, in which the two forms are in *dynamical equilibrium* with each other at each temperature; so that, when the latter is more or less completely established, the composition of the phase must appear to be continually-variable with the temperature; — in the same way as this seems also to have been observed in the case of the equally cubic α- and β-forms of *manganese*.

In this way, therefore, the metal *rhodium*, with its "dynamic allotropism", evidently represents *the natural transition-term* between the *first* tetrade of elements in the 8th group, — with its conspicuous "static" allotropism —,

and the *latter* one with its complete lack of reversible changes in the solid state.

From all this, it thus becomes evident, that also with respect to the phenomena here studied, the eighth group of the periodic system in its present form really constitutes *an entity*, with the rigorous character of true systematical uniformity.

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Anatomy. — *Arrhénoblastomes et changement de sexe.* Par G. KREDIET.

(Communicated at the meeting of June 29, 1940.)

Le changement de sexe était déjà connu du temps des Romains. TACITE mentionne que l'on devait signaler chaque année le nombre des poules changées en coqs. De même, au Moyen Age, ce phénomène était parfaitement connu et on le considérait comme une incarnation de démon; il en résultait que la personne, qui subissait ce changement ou chez laquelle un animal présentait cette métamorphose, était brulée vive. A l'heure actuelle, il y a peu de cultivateurs qui ignorent que les vieilles poules peuvent changer de sexe. On considère cependant encore ce changement comme une espèce de prodige. Est moins connu le fait que la femme et les animaux domestiques femelles peuvent prendre le sexe masculin; les cas ne sont pas rares, en effet dans la littérature on en signale déjà une quantité.

La signification biologique de ce phénomène est restée longtemps ignorée. Grâce à GOLDSCHMIDT les connaissances à ce sujet sont devenues plus étendues. Cet auteur profita de l'expérience des éleveurs-amateurs de papillons, qui avaient observé que par l'accouplement de différentes races de Lymantria dispar on pouvait obtenir des papillons, dont le sexe était intermédiaire entre celui des individus mâles et femelles. Il a systématiquement poussé plus loin ces expériences, les a contrôlées au point de vue génétique et histologique et est arrivé à la conclusion qu'au moment de la fécondation l'intersexualité, ainsi que son degré, sont déjà déterminés. Deux notions importantes se sont fait jour au cours de ces observations, notamment celle de l'épistase et celle du moment où la transformation de sexe doit avoir lieu. Lors de la détermination du sexe des facteurs mâle M et femelle F interviennent. Chez un individu mâle le facteur M est le plus fort et chez un individu femelle c'est le facteur F. Si un individu doit être mâle pur, le facteur M doit être tellement puissant que le facteur F n'est plus en état d'exercer une influence quelconque sur le sexe et ses attributs. Chez un animal femelle c'est l'inverse. Cette prédominance ou épistase doit avoir une certaine grandeur, en d'autres termes doit passer un minimum, pour pouvoir dominer complètement l'autre sexe. Si la différence est plus petite que le minimum épistatique, alors les bases sont jetées pour l'édification d'un intersexué. Il est cependant un fait remarquable, comme l'examen histologique l'a d'ailleurs prouvé, qu'en pareil cas l'évolution du sexe commence par ce que d'après le mécanisme X—2X doit être le sexe, mais qu'à un certain moment de la différenciation cette évolution prend la direction hétérosexuelle et se développe ultérieurement en ce sens. On peut facilement s'en faire une idée avec l'aide du schéma ci-joint. F représente la courbe des facteurs fémelles; M1—M9 les facteurs mâles

d'après leurs différentes possibilités. M9—F figure les facteurs de l'animal mâle. Entre M9 et F il y a une prépondérance suffisante de M9, de sorte

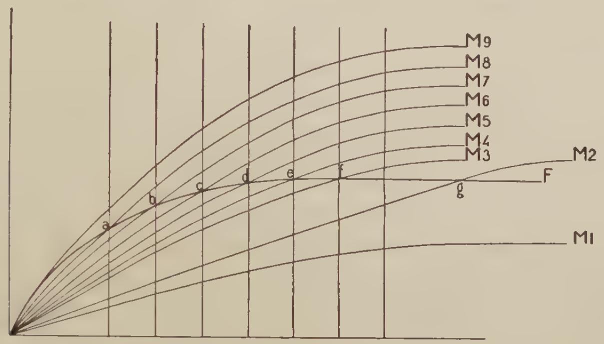

Fig. 1.

que les facteurs mâles sont toujours les plus puissants. En sens inverse la même situation se réalise pour M1—F, qui représentent donc les courbes d'un animal femelle. Toutes les autres combinaisons de M avec F figurent la situation chez les intersexués. Aux points a, b, c, d, e, f et g les facteurs M de l'animal originairement femelle deviennent plus puissants et le changement de sexe a donc lieu. A l'endroit M8—F M est plus puissant qu'au point M2—F; l'influence de F est au contraire plus faible.

A l'aide de ce schéma il est possible d'interpréter les diverses formes que peut revêtir l'intersexualité chez l'homme et chez l'animal. La difficulté de pouvoir rechercher les causes de l'intersexualité chez les animaux supérieurs réside dans le fait que le changement de sexe s'opère dans la plupart des cas pendant la vie intrautérine, donc avant la naissance et qu'il est par conséquent difficile de trouver des individus propres à démontrer le processus de ce changement. Dans la majorité des cas on a du se contenter de déduire des phénomènes observés le stade de l'évolution sexuelle, dans lequel le changement avait eu lieu. A ce point de vue, les cas où le changement a pu être constaté immédiatement après la naissance revêtent une importance particulière. Le plus souvent les symptômes s'y rapportant peuvent être observés aux gonades de porcelets nouveaux-nés, rarement chez la chèvre et exceptionellement chez l'homme (cas de Blair Bell).

A cet égard les oiseaux fournissent un matériel intéressant. Ils montrent de la façon la plus nette le changement de sexe. Il est possible qu'en de pareils cas on ait à faire à des intersexués ordinaires, mais aussi à des animaux porteurs de tumeurs de l'ovaire, du péritoine ou de la couche corticale des glandes surrénales. Chez l'homme on a observé que les cas de changement de sexe, s'opérant après la naissance sont dans la grande majorité des cas caractérisés par la présence de tumeurs de l'ovaire ou de la couche corticale des surrénales. A la suite de cette constatation, le problème de l'intersexualité s'est étendu de façon telle que de nouvelles

difficultés d'interprétation ont apparu. On était tenté de croire que les tumeurs pourraient être la cause du changement des caractères sexuels, surtout lorsqu'après leur extirpation, on constata un retour au sexe primitif. Ce raisonnement était admissible, car d'après les expériences mémorables de STEINACH, LIPSCHÜTZ et autres, il a été prouvé, qu'il était possible de masculiniser ou de féminiser un castrat par la transplantation de gonades hétérosexuelles ou par l'injection des hormones hétérosexuelles. L'intersexualité expérimentale est donc un fait acquis; elle serait semblable à celle provoquée par des tumeurs ovariennes ou surrénales, parce qu'on admet que celles-ci sécrètent des hormones hétérosexuelles. Ces tumeurs seraient donc primaires. En opposition avec cette manière de voir, il y en a une autre, qui met sur le même pied les intersexués ordinaires et les cas de masculinisation avec tumeur et qui admet que le changement de sexe est la cause et la tumeur la consequence; cette dernière devant être assimilée à une gonade intersexuee.

Il n'est assurément pas sans intérêt d'examiner lequel des deux points de vue est le plus admissible. Je voudrais essayer par une étude comparative de la masculinisation chez l'homme et chez l'animal, de rechercher s'il s'agit d'intersexués ou de masculinisation par tumeur. Les cas humains ne me sont connus, cela va de soi, que par la littérature; personellement j'ai observé le changement du sexe féminin en masculin chez deux chèvres, quelques poules et canes, un cheval et une vache. Les symptômes furent en résumé les suivants: Une des chèvres avait été en chaleurs et saillie par un bouc. Par après l'animal n'avait plus été en chaleurs, mais n'avait pas non plus été fécondé. La tête changea, reçut un fort revêtement pileux, devint plus large; l'expression des yeux devint plus méchante et, chose curieuse, le pis s'hypertrophia fortement et donna même $1\frac{1}{2}$ Litres de lait normal par jour. Plus tard la sécrétion lactée diminua et le pis devint plus petit. La conduite de l'animal changea fortement. D'abord maniable comme une chèvre, il devint plus tard intraitable et même dangereux. Auprès d'une chèvre, il montrait un vif appétit sexuel mâle; avec des boucs il commençait immédiatement à se battre. A l'autopsie on découvrit deux ovariotestes. La seconde chèvre était encore jeune quand je l'achetai. Elle était en piètre état d'entretien. Du fait que, peu de temps après, l'état général s'améliorait et que le pis commençait à sécréter du lait, le soigneur avait conclu que l'animal avait été fécondé. De même que chez l'animal précédent l'appareil génital externe était resté femelle et le pis se met à sécréter du lait. Cet organe n'atteignit cependant pas les mêmes proportions que chez le premier animal, la sécrétion lactée ne se maintint pas aussi longtemps. Entretemps l'extérieur s'était rapproché de celui d'un bouc; auprès d'une chèvre l'animal ne restait pas indifférent, mais se comportait, surtout auprès de chèvres en chaleurs, d'une façon fort mâle. Vis á vis de boucs, il engageait immédiatement le combat. Ce processus de changement s'accomplit au cours de 4 mois de temps. A l'autopsie, on trouva également deux ovariotestes. Chez aucune de ces deux chèvres, il n'existait une hypertrophie du clitoris.

Le cheval, une jument, fut remarqué dans une infirmerie militaire à cause de ses manières d'étalon, qu'il n'avait pas montrées auparavant. Il saillissait les juments, prenait une attitude d'entier, acquit l'expression de regard typique pour le mâle et laissait l'impression que les ganaches et de ce fait la tête étaient devenues plus lourdes. Le vétérinaire militaire traitant était d'avis que le clitoris était plus développé que normalement; mais à la clinique de la faculté vétérinaire d'Utrecht, où le cheval fut amené, on était d'avis que le développement du clitoris n'avait rien d'anormal. Il est probable que le clitoris était en train de se développer, mais l'animal n'ayant pas été vu antérieurement par celui qui l'examina, celui-ci n'a pas osé conclure que l'organe était trop développé. Au cours de l'opération, on trouva un ovaire gauche normal et un ovaire droit fortement agrandi par une tumeur. Cette tumeur fut examinée par ROBERT MEYER de Berlin, qui diagnostiqua un cystadenome de la partie tubulaire, mais il est à remarquer que de nombreux kystes possédaient une structure rappelant celle d'un follicule, de sorte que le diagnostic folliculome tubulaire aurait également convenu.

Les cas de masculinisation que j'ai observés chez les poules et une fois chez une cane, ont été décrits in extenso dans l'Archiv für Gynaekologie, Bd. 158, 1934, 22—53. J'ai pu les diviser en 4 groupes, présentant en ordre croissant des signes de masculinisation de plus en plus prononcés et allant de pair avec des changements ovariens parallèles. Deux données capitales se dégageaient de cette étude. En premier lieu, que les changements morphologiques externes étaient identiques chez les animaux d'un même groupe, de sorte que seule l'autopsie pouvait déterminer s'il s'agissait d'une tumeur ovarienne ou d'un ovaire évoluant dans les sens d'un ovario-testis ou d'un testicule. En second lieu, que quelle que soit la constatation faite à l'autopsie, les tumeurs aussi bien que les changements testiculoïdes émanaient des mêmes cellules embryonnaires; les cellules-mères de ces néoformations étaient les cellules médullaires et corticales, qui n'interviennent pas dans la formation folliculaire.

La vache, qui changea en taureau, avait été une fois en chaleurs; elle avait été saillie, mais n'avait pas été fécondée. A part les organes génitaux externes, elle acquit tous les attributs de mâle. A l'autopsie, on découvrit deux testicules. En ce qui concerne la genèse de la masculinisation, les recherches effectuées ne fournirent pas les résultats qu'on aurait pu espérer chez un animal qui avait été suivi de si près.

A part ces cas, que j'ai pu observer au cours d'une vingtaine d'années, il m'a été possible d'examiner quelquefois des préparations microscopiques de gonades de porcs intersexués en bas âge, qui montraient un processus de changement de sexe encore en cours. Presque sans exception, la transformation s'opérait du sexe féminin vers le sexe masculin.

Aussi bien dans les cas signalés dans la littérature, que dans les cas mentionnés plus haut, il est possible de distinguer deux groupes, que j'ai désignés sous les noms de *intersexués physiologiques* et *intersexués patho-*

logiques. Les derniers sont des individus, chez lesquels la masculinisation s'accompagne de la formation d'une tumeur, les premiers étaient des animaux, chez lesquels la glande sexuelle subit des changements dans le sens du sexe opposé.

Les études les plus étendues dans le domaine des tumeurs masculinisantes, se rapportent à tous les cas observés chez l'homme. En principe les symptômes sont identiques à ceux présentés par les animaux, de même les tumeurs peuvent, en ce qui concerne leur genèse, être mises sur le même pied que celles des animaux, de sorte que les éléments nécessaires à une étude comparée existent.

De toutes les tumeurs trouvées en cours des années dans les ovaires, celles qui sont en rapport avec la masculinisation ne constituent qu'une infime minorité. Elles ne sont pas caractérisées par une structure spéciale, de sorte que les différents chercheurs ont posé les diagnostics les plus divers. ROBERT MEIJER a eu le grand mérite de les avoir regardées toutes sous un angle unique et de les avoir désignées par un seul nom. Il les nomme *arrhénoblastomes*, mais il se trouve obligé, vu leurs aspects divers, de faire différents groupes, qui ont cependant de commun que ce sont *des tumeurs épithéliales et qu'elles peuvent être ramenées à des cellules génitales à potentiel embryonnaire*. En dehors des arrhénoblastomes, il existe encore d'autres tumeurs satisfaisant à ces exigences. Ce sont les tumeurs à cellules de granuleuse ou folliculomes et les carcinomes à grandes cellules, séminomes ou disgerminomes, comme les appelle ROBERT MEIJER. D'après ce chercheur, toutes les deux n'ont cependant rien à voir avec la masculinisation et elles constituent donc des tumeurs possédant d'autres propriétés; ainsi les premières p. ex. conduiraient à l'hyperféminisation.

Les arrhénoblastomes sont subdivisés comme suit:

a. l'adénome testiculaire tubulaire de l'ovaire (PICK), qui par sa structure rappelle le plus celle du testicule et qui, chose étrange, ne serait pas toujours accompagné de masculinisation

b. les formes atypiques caractérisés par des proliférations épithéliales diffuses, des cordons et des canalicules; elles peuvent contenir des kystes, même des kystes muqueux et ont une tendance aux hémorrhagies, aux liquéfactions et à la nécrose

c. les formes intermédiaires entre *a* et *b*, qui possèdent des masses épithéliales atypiques, peuvent renfermer les canalicules et des cordons évoluant dans le sens testiculaire.

Les tumeurs à cellules de granuleuse sont considérées par ROBERT MEIJER comme étant spécifiquement féminines et pouvant donner lieu à l'accroissement et même à la sécrétion des mamelles. On connait néanmoins dans la littérature des cas où elles sont considérées comme étant la cause de la masculinisation qui les accompagne. BERGSTRAND a été le premier à attirer l'attention sur ce fait. Il signale six cas, dont 3 se rapportaient à des tumeurs ovariennes composées. Un de ces 6 cas a été reconnu par BERNER comme étant un adénome testiculaire tubulaire, alors que d'autre part

ROBERT MEIJER a porté pour un autre le diagnostic d'arrhénoblastome. SCHILLER a également examiné un cas de masculinisation, où il trouva une tumeur à cellules de granuleuse, plus tard cependant il est revenu sur ce diagnostic. SOETIDJO HARDJOSOEKATMO mentionne dans „Geneeskundig Tijdschrift voor Nederlands Indië", au cours de la description d'un cas d'arrhénoblastome malin de l'ovaire, que EERLAND et VOS ont posé le diagnostic de tumeur à cellules de granuleuse dans un cas, qui auparavant avait été admis comme arrhénoblastome. La masculinisation que j'ai observée chez une jument était accompagnée d'une tumeur de l'ovaire que ROBERT MEIJER appela cystadénoma partim tubulare et dans laquelle il y avait des grands et des petits kystes, dont la paroi rappelle celle d'un follicule et de laquelle SALOMON, qui travaille dans le laboratoire de PEYRON, dit qu'elle ressemble aux tumeurs microkystiques relativement fréquentes chez la jument, considérées d'ordinaire comme des folliculomes (les cas de masculinisation sont cependant très rares chez le cheval). A l'opinion de ROBERT MEIJER, d'après laquelle les tumeurs à cellules de granuleuse sont spécifiquement féminines, on doit donc apporter ce correctif qu'il peut exister dans les cas de masculinisation des circonstances, dans lesquelles elles peuvent être rencontrées. On ne peut dénier que de la sorte la spécificité de l'arrhénoblastome est battue en brèche. Quelle est à ce point la situation des carcinomes à grandes cellules? Ces tumeurs se rencontrent chez des animaux aussi bien mâles, que femelles et intersexués. Elles sont connues comme étant des tumeurs des gonades chez les intersexués. Elles portent encore d'autres noms, dont les plus connus sont: séminome, épithéliome séminifère, sarcome alvéolaire, disgerminome. MASSON et avec lui CHENOT fut le premier à reconnaître un épithéliome séminifère dans un ovaire. ROBERT MEIJER était d'avis que l'appellation disgerminome était adéquate, parce que la tumeur ne procéderait pas des cellules des tubes séminifères, mais bien d'un épithélium germinatif neutre, qui serait de qualité inférieure (unterwertig), donc disgerminatif. Ce que cet épithélium aurait perdu en potentiel, il l'aurait gagné en puissance proliférative. H. O. NEUMANN affirme que ces tumeurs se rencontrent le plus souvent dans les gonades d'intersexués, alors que MOSZKOWICZ fait remarquer que si l'on ne les rencontre pas chez des intersexués, ce sera du moins chez des individus présentant des signes d'hypogénitalisme.

Les carcinomes à grandes cellules seraient donc des tumeurs tout autres que les arrhénoblastomes. ROBERT MEIJER signale les différences suivantes: „Während diese auf ein spezifisch männlich gerichtete Anlage oder Verwandlung der Zellen durch ihre besondere Wirkung hin erweisen, so kommt ähnliches nicht bei den grosszelligen Carcinomen vor. Sie erscheinen nur als Begleiter der angeborenen Zwittrigkeit, während umgekehrt die Arrhenoblastomen nur bei Personen bekannt sind, die von Haus aus nicht zwittrig erscheinen, sondern es im gewissen Grade erst werden und zwar unter dem Einfluss der Geschwulst." D'après le même auteur, les carcinomes à grandes cellules n'ont, au point de vue morphogénétique rien de

commun avec une „zwittrige Anlage", „sondern nur in der Verbindung mit der Abartung von sexueller Epithelanlage, also mit Asexualität der Zellen und zwar in einer frühembryonalen Zeit, bevor die normale geschlechtliche Differenzierung der Keimepithelien erfolgt. Dieses ist wahrscheinlich, weil sonst kaum eine so weitgehende Aehnlichkeit, ja Gleichheit der Geschwulst bei beiden Geschlechtern sich ergeben würde."

Il est difficile de concilier cette opinion avec la réalité de la formation du sexe. Ces tumeurs seraient donc issues de cellules embryonnaires asexuées, rien que parce qu'elles sont les mêmes chez les individus à sexe normal et chez les intersexués. Pour ce motif ces cellules devraient être originaires d'une époque précédant la différenciation des épithéliums germinatifs (cellules-mères des cellules germinatives). En réalité les choses ne se passent pas ainsi. Il n'existe pas de stade asexuel, car lors de la fécondation le sexe (éventuellement l'intersexe) est déterminé et toutes les cellules de l'individu, aussi bien les somatiques que les germinatives. possèderont ce même sexe. Le fait qu'il existe un stade embryonnaire, où le sexe de l'individu ne peut pas encore être reconnu, est uniquement la conséquence de l'insuffisance de différentiation des organes génitaux et cela ne saurait rien changer au fait que l'embryon avec toutes ces cellules possède déjà un sexe déterminé. L'interprétation donnée par ROBERT MEIJER de la morphogenèse des carcinomes à grandes cellules, n'est pas exacte en ce qui concerne les relations de ces tumeurs avec le sexe de l'individu qui en est porteur.

La différence entre les carcinomes à grandes cellules et les arrhénoblastomes résiderait dans le fait que les derniers sont à réduire à une prédisposition ou à une transformation de cellules embryonnaires dans le sens spécifiquement masculin, tandis que les premiers ne constitueraient qu'un phénomène accompagnant l'intersexualité congénitale. Qu'est-ce qu'une prédisposition à sens masculin chez un individu femelle? Est-ce une partie du corps ou de l'ovaire qui a changé ou qui est en train de changer de sexe? Ou est-ce une partie issue d'une cellule qui a perdu un des X-chromosomes? On sait que les animaux peuvent changer de sexe; cela est la conséquence de la détermination lors de la fécondation dans le cas d'un intersexué, qui ne peut valoir jamais pour une partie de l'individu. En ce qui concerne la perte d'un X-chromosome, on peut faire remarquer que la masculinisation est également connue chez les oiseaux, de telle sorte que l'on devrait parler ici d'augmentation d'un X-chromosome! La prédisposition à sens masculin, aussi bien que l'asexualité des cellules représentent deux notions, qui n'ont aucune signification pour la genèse des arrhénoblastomes et des carcinomes à grandes cellules.

Si l'on examine de près les arrhénoblastomes, on arrive à la conclusion qu'il ne s'agit pas d'une tumeur à structure spécifique. Ceci est confirmé par la classification en groupes différents, dans lesquels sont rangées des tumeurs de forme et de structure fort diverses. Toutes ces tumeurs ont 3 propriétés de commun:

a. on les trouve dans les cas de masculinisation
b. elles sont de nature épithéliale
c. elles procèdent de cellules à potentiel embryonnaire et génital.

Une tumeur à cellules de granuleuse, qui serait spécifiquement féminine, mais qu'on rencontrerait chez un individu masculinisé répondrait aussi à ces trois conditions et constituerait donc également un arrhénoblastome. BENECKE (cité par HARDJOSOEKATMO), du laboratoire de ROBERT MEIJER range les carcinomes à grandes cellules parmi les tumeurs masculinisantes. de sorte qu'on peut se demander s'il n'est pas préférable de mettre une fin à cette confusion en nommant arrhénoblastomes toutes les tumeurs rencontrées dans les cas de masculinisation ou se trouvant en rapport avec ce processus. A ce groupe, représentant une notion fonctionnelle, peuvent alors appartenir toutes les tumeurs qui répondent aux trois conditions précitées, quel que soit leur caractère morphologique ou quel que soit le diagnostic que l'anatomo-pathologiste pense devoir établir.

En présence de la multiplicité des tumeurs, on peut se poser la question si ces tumeurs peuvent bien être une cause de masculinisation. Il est tellement difficile d'admettre que des structures si dissemblables auraient toutes la même signification. Ne vaudrait-il pas mieux de considérer les tumeurs comme des phénomènes d'accompagnement et de rechercher la cause de la masculinisation chez l'individu même? Quand on raisonne de cette façon, on est frappé par la grande ressemblance que présentent les intersexués physiologiques et les intersexués pathologiques. Chez aucune espèce animale cette ressemblance est aussi nette que chez les oiseaux. Les intersexués physiologiques mammifères ont accompli leur transformation sexuelle au stade intrautérin; on ne peut guère l'observer, mais on peut la supposer avec de grandes chances de probabilité. S'agit-il au contraire d'intersexués pathologiques, alors ce changement est constaté dans le courant de la vie. Chez les oiseaux ce processus s'accomplit dans les deux cas après l'éclosion et il est caractérisé par des phénomènes tout à fait identiques, de sorte que ce n'est qu'à l'autopsie qu'on peut s'assurer s'il s'agit de l'une ou de l'autre espèce d'intersexués. Dans les deux cas on trouve dans l'ovaire des néoformations, qui procèdent des mêmes cellules embryonnaires. Il s'en suit que le parenchyme tumoral et les tubes testiculaires, dans lesquels peut même s'accomplir une spermatogenèse parfaite, ont la même origine, de sorte que les ovariotestes, eventuellement les testicules et les arrhénoblastomes sont équivalents. En ce qui concerne la morphogenèse, il n'existe donc pas de différences de principe entre les intersexués physiologiques et pathologiques. Seul le produit des cellules embryonnaires à potentiel génital est différent. On peut donc conclure des faits observés que dans les deux cas le changement de sexe est la cause des néoformations, qui se produisent dans l'ovaire, mais on ne saurait dire pourquoi les cellules latentes embryonnaires, qui ont été poussées à se multiplier, donnent naissance dans l'un cas à des éléments plutôt normaux et dans l'autre à des éléments anormaux. Il apparait de toute évidence

qu'ici aussi il faut penser à une cause d'ordre général, parce que non seulement les cellules médullaires masculines primitives, mais aussi les cellules corticales, considérées comme féminines, participent au processus.

Dans cet ordre d'idées, la direction dans laquelle s'engage l'évolution de la tumeur, n'a guère d'importance. Cette direction dépend de facteurs inconnus jusqu'à présent et qui vraisemblablement sont en rapport étroit avec le processus de changement de sexe et en dépendent peut-être. Je voudrais considérer ces facteurs, aussi bien que les tumeurs comme des phénomènes d'accompagnement, de sorte que l'on ne doit pas rechercher l'élément primaire dans la tumeur ou dans la cause qui préside à sa formation, mais bien dans le changement de sexe.

A la rigueur de pareilles tumeurs ne pourraient pas être appelées arrhénoblastomes et l'on pourrait supprimer cette dénomination ou la remplacer par une autre, mais si l'on veut quand même conserver ce terme, proposé par ROBERT MEIJER, en raison de grand mérite de ses études sur les cas de masculinisation avec tumeur, dans ce cas il faut qu'on y attache un sens quelque peu différent. Le terme arrhénoblastome devrait donc désigner les tumeurs formées lors de masculinisation par des cellules de l'ovaire à potentiel embryonnaire et génital; la notion étiologique doit donc être totalement supprimée.

Tout le monde ne se déclarera pas d'accord avec cette manière de voir, parce qu'en démontrant une morphogenèse identique des néoformations de l'ovaire chez les intersexuès physiologiques et pathologiques on n'a pas détruit tous les arguments en faveur de la théorie étiologique des arrhénoblastomes. Il existe d'ailleurs encore deux arguments qui à première vue semblent fortement plaider en faveur de cette opinion. Le premier se base sur le fait que chez les femmes, auxquelles on enlève la tumeur, les caractères féminins réapparaissent. On connaît même des femmes pareilles qui par après ont encore eu des enfants. Le second argument réside dans la présence de tumeurs de la couche corticale des surrénales dans des cas de masculinisation. De tels blastomes ne peuvent pourtant pas être issus des cellules médullaires ou corticales de l'ovaire, en sorte que les considérations émises plus haut ne sauraient y être appliquées.

Et malgré tout, je ne pense pas que ces deux arguments sont en contradiction avec l'opinion que le changement de sexe est primaire et que la tumeur est secondaire.

L'opinion d'après laquelle le retour des caractères féminins serait la conséquence de disparition des hormones hétérosexuelles après l'enlèvement de la tumeur, est fort attrayante, parce que tellement simple et s'accordant si facilement avec les faits observés. Je ne nierai pas que la sécrétion interne de la tumeur joue un rôle important, mais, tout aussi bien que la tumeur, cette sécrétion n'est qu'une conséquence. A mon avis le défaut de cette opinion réside dans une figuration erronnée de l'apparition des caractères sexuels secondaires. Ceux-ci ne sont pas uniquement le résultat de l'influence des hormones gonadales. Il existe encore une autre

influence qu'on néglige d'ordinaire. Il faut prendre comme point de départ le moment de la fécondation où le sexe est déterminé avec tous ses attributs. Chaque cellule procédant de l'ovule fécondé a le même équipement chromosomique que celui-ci et est donc du même sexe. On appelle cela le sexe cellulaire ou somatique, parce que tout le corps le possède. Si un individu naît sans glandes génitales, il possède néanmoins un type masculin ou féminin. Pareil animal n'est pas asexué, comme on le pense souvent, mais il lui manque les témoins du sexe sous forme de glandes génitales. Les raisons de ce sexe somatique résident dans les facteurs chromosomiques, présents dans chaque cellule et exerçant leur influence toute la vie durant. Elles sont primaires et sont responsables de l'ébauche des caractères sexuels secondaires. A la puberté se fait sentir la grande influence des hormones gonadales, qui réalisent en peu de temps le parachèvement total du type sexuel et le maintien de cet état pendant toute la durée de la vie sexuelle. Lors de la castration ces influences sont supprimées, mais l'action des facteurs chromosomiques persiste. Après avoir été châtré, un animal n'est pas devenu asexué, il est uniquement un individu sans glandes génitales.

Par le fait qu'un arrhénoblastome peut provoquer si rapidement l'évolution des caractères secondaires mâles, on a toutes les raisons d'admettre qu'il secrète des hormones hétérosexuelles, capables d'extérioriser les caractères masculins pouvant encore apparaître dans un individu jusque là féminin. Après extirpation de la tumeur, la sécrétion interne est supprimée, les caractères masculins formés rapidement disparaissent tout aussi vite et les attributs féminins réapparaissent. Les caractères masculins, qui ne peuvent complètement rétrocéder, se maintiennent, p. ex. la pomme d'Adam trop développée. Souvent aussi il est fait mention d'une persistance partielle de la voix grave, vestige typique de la masculinisation.

Après avoir extirpé la tumeur en même temps que l'ovaire, qui en est le porteur, il persiste encore l'ovaire de l'autre côté et l'on peut se demander quelle sera son influence sur l'organisme. Pour ceux qui attachent une signification primaire à la tumeur, la réponse est simple. L'individu est à nouveau devenu féminin et l'a d'ailleurs toujours été; donc l'ovaire continuera son activité en ce sens. Pour ceux qui considèrent ces patients comme des intersexués, la question se pose autrement; ils doivent en effet tâcher de trouver une solution au moyen de la théorie de GOLDSCHMIDT. A ce propos, j'attire l'attention sur les facteurs M et F qui déterminent le sexe. Le cas sus-visé avec changement de sexe après la naissance est déterminé par M2—F (fig. 1). Depuis longtemps F et M2 s'en vont très peu l'un de l'autre et même après le changement de sexe, la différence entre les deux n'est pas très grande. Il n'existe qu'une faible intersexualité; même après que M2 est devenu le plus fort. F exercera encore une grande influence dans l'organisme. Le renforcement de M2 par les hormones de la tumeur disparaît après l'extirpation.

L'ovaire restant est une glande sexuelle dans un organisme intersexué.

Afin de pouvoir juger de la signification des gonades dans ces circonstances extraordinaires, je me suis occupé, il y a une dizaine d'années, d'enlever les gonades à un grand nombre d'intersexués. J'ai pu ainsi observer quels sont les caractères disparaissant de ce qui restait de l'appareil génital. Il m'a semblé qu'on pouvait mettre les résultats obtenus sur le même pied que ceux obtenus après castration d'animaux normaux au point de vue sexuel. Quelles que fussent les gonades enlevées chez les intersexués (ovariotestes ou testicules), il est certain qu'elles assuraient le développement sexuel de l'appareil génital pendant la vie sexuelle. D'ordinaire cet appareil génital se compose de parties mâles et femelles, de telle sorte que p. ex. les testicules assuraient le développement total de la matrice. On a supposé qu'une hormone intersexuelle assurait le développement du caractère sexuel chez les hermaphrodites. De REGT n'est pas parvenu à mettre en évidence la présence de cette hormone gonadale dans l'urine des porcs intersexués, mais il trouva des hormones mâles et femelles dans des proportions tout autres que chez des individus normaux. BUTENANDT est parvenu plus tard à préparer une hormone intersexuelle.

Il est donc possible que l'ovaire de l'individu masculinisé se comporte comme la glande génitale d'un intersexué et qu'il sécrète donc des hormones testiculaires et ovariennes dans des proportions autres qu'à l'état normal. Je ne puis pas cependant attacher une grande importance à ces faits, car KOCH a démontré qu'au cours d'une période de quatre semaines, il y a chez une femme des époques où elle excrète dans l'urine des quantités plus grandes d'hormone masculine que d'hormone féminine.

L'observation suivante semble être plus importante. Il s'agit notamment de la comparaison que j'ai faite entre les possibilités de fécondation chez un animal femelle et chez un animal intersexué. Chez le premier les possibilités dépendent des chaleurs, de l'ovulation et de la capacité pour l'appareil génital de recevoir les spermatozoides et de préparer la voie vers l'ovule arrivé à maturité. Chez le second les organes génitaux ne sont d'ordinaire pas à même de remplir la dernière condition, de sorte que la comparaison doit se limiter aux chaleurs et à l'ovulation. Chez des intersexués, j'ai pu constater quelquefois des chaleurs manifestés alternant souvent avec une excitation sexuelle mâle. J'ai aussi trouvé deux générations successives de corps jaunes; chez un animal dans deux ovariotestes et chez un autre dans un ovariotestis et dans un ovaire. Ils étaient tout à fait identiques à ceux qu'on trouve dans un ovaire normal.

S'il était impossible pour un intersexué de produire de l'ovulation et d'entrer en chaleurs, ces deux propriétés seraient caractéristiques pour un individu femelle; mais comme tel n'est pas le cas, la thèse d'après laquelle sont intersexués les individus se masculinisant tout en présentant un développement de tumeur, n'est pas démolie. Par conséquent l'argument du retour des caractères féminins après enlèvement de la tumeur ne saurait être jeté dans la balance en défaveur de cette thèse et cela avec d'autant plus de raison que ces individus possèdent une faible intersexualité (M2—F).

La seconde objection contre l'état intersexué des individus, dont la masculinisation est caractérisée par le développement d'une tumeur, se rapporte à ceux qui sont porteurs d'une tumeur de la couche corticale de surrénale, formée de cellules procédant de cette substance corticale. A première vue, il n'existe pas ici de cellules embryonnaires à potentiel génital, comme les cellules médullaires et corticales de l'ovaire. On a cependant affaire ici au blastome d'un tissu fournissant une sécrétion interne et qui, comme on le sait d'ailleurs, peut après la castration sécréter d'une façon vicariante les hormones produites à l'état normal par la gonade. On peut donc admettre, sans risquer de se tromper, que la sécrétion interne de la tumeur est à même de réaliser la transformation des caractères sexuels secondaires.

Afin de pouvoir rechercher la nature et la signification des tumeurs de la couche corticale des surrénales et de pouvoir les comparer avec les arrhénoblastomes, il est d'importance capitale de connaître les propriétés des cellules dont précèdent les tumeurs. Il est surtout important de s'enquérir de l'existence possible dans ces cellules de la couche corticale du potentiel présent dans les cellules ovariennes embryonnaires.

Le développement de cet organe doit fournir les indications requises. On sait que la surrénale procède de deux ébauches. La partie médullaire (organe suprarénal) dérive du système nerveux autonome, des sympathogonies procédant des cellules du cordon-limite. La partie corticale (organe interrénal) a son origine dans la partie orale de l'épithélium péritonéal qui donne naissance aux gonades. La partie corticale de la surrénale et les gonades possèdent donc un blastème commun. La différence entre les deux réside dans le fait que l'épithélium péritonéal, dont sont issus les testicules et les ovaires sous l'influence des gonocytes primaires, devient épithélium germinatif, tandis que cette transformation ne s'opère pas pour le tissu-mère de la couche corticale de la surrénale. Il est possible que des gonocytes primaires extrarégionaux puissent jouer un rôle ici, mais il est alors difficile de concevoir pourquoi l'épithélium péritonéal ne devient pas épithélium germinatif et pourquoi l'organe qui en est issu ne devient pas une glande sexuelle. Pour ce motif il est préférable de rechercher quelles sont les propriétés des cellules de l'épithélium péritonéal. Au point de vue phylogénétique, c'est un tissu remplissant 3 fonctions fondamentales: fonction de revêtement, fonction sécrétoire et fonction de formation de cellules germinatives. La première de ces fonctions est conservée chez les animaux supérieurs, les deux autres ont été transmises à des organes distincts: reins et glandes génitales, mais il y a des indications montrant que la fonction génitale n'est pas entièrement abolie. Je ne vise pas ici en premier lieu les expériences d'ovariotomie chez des oiseaux, à la suite desquelles un nouvel ovaire se serait formé à partir de l'épithélium péritonéal, parce que d'autres expériences contredisent ces résultats. En de pareils cas, il est toujours possible qu'un reliquat d'ovaire ait été le point de départ du nouvel organe. Mais je voudrais attirer l'attention sur quel-

ques animaux caractérisés par la présence de testicules multiples disséminés sur tout le péritoine. Moi-même j'ai eu l'occasion d'examiner pareil porc. Il n'est pas impossible que des gonocytes primaires erratiques ont d'abord changé ici l'épithélium péritonéal en épithélium germinatif. Le fait que cette transformation s'est opérée uniquement dans le péritoine et pas en d'autres endroits où des gonocytes errants auront probablement aussi échoué, plaide en faveur de la prédilection du péritoine et du maintien de ses propriétés primitives. Je veux au surplus rappeler le cas d'une poule masculinisée qui possédait une multitude de tumeurs, des papillo-cysto-carcinomes issus tous de l'épithélium péritonéal. J'insiste tout spécialement sur le cas de ce porc et de cette poule, parce qu'ils constituent des expériences de la nature, ayant comme point de départ les propriétés de l'épithélium péritonéal. On ne sait pas sous quelles influences le potentiel latent se manifeste à nouveau, mais ces deux cas prouvent que ce potentiel continue à exister.

Ces tumeurs de la poule issues de l'épithélium péritonéal ont comme origine des cellules à potentiel embryonnaire génital, sont rencontrées dans une cas de masculinisation et répondent par conséquent à la définition des arrhénoblastomes. Il n'y a donc aucune raison pour ne pas classer ces papillo-cysto-carcinomes parmi cette catégorie.

Il existe donc une parenté cellulaire entre les gonades et la couche corticale de la surrénale, donc également entre les tumeurs procédant des cellules médullaires et corticales de l'ovaire et celles issues de la couche corticale de la surrénale. Il devient donc évident que les tumeurs de la couche corticale de la surrénale dans des cas de masculinisation se rapprochent sensiblement, en ce qui concerne leur genèse, des arrhénoblastomes de l'ovaire. Dans ces conditions, il n'est pas étonnant qu'elles puissent posséder semblable sécrétion interne. Dans ces deux cas, aussi bien que dans les gonades, cette sécrétion est la conséquence des facteurs génitaux qui priment dans l'organisme, tout en tenant évidemment compte de la différence minime entre F et M dans les cas où la masculinisation se constate dans le courant de la vie (cas M2—F). La sécrétion hormonale d'une tumeur de la couche corticale de la surrénale, prenant la direction masculine après le changement de sexe, n'empêche pas d'admettre que non plus la tumeur soit primaire, mais bien le changement de sexe.

Il me semble que toutes les objections soulevées contre la nature intersexuelle des individus changeant de sexe au cours de l'évolution des tumeurs de l'ovaire ou de la couche corticale de la surrénale, tombent quand on analyse la genèse des tumeurs et qu'on tient compte de la formation de l'intersexe. Je partage l'avis d'HALBAN, qui subdivisait les ovules en féminins, masculins et hermaphrodites et qui proclamait que les femmes devenant homme devaient posséder une prédisposition hermaphroditique. C'est avec raison que cet auteur attribue un rôle protecteur à la sécrétion interne influençant le développement des caractères sexuels secondaires.

Physics. — *On the buckling of a thin-walled circular tube loaded by pure bending.* I. By C. B. BIEZENO and J. J. KOCH.

(Communicated at the meeting of June 29, 1940.)

1. *Introduction.* If a thin-walled circular tube in his end-sections is loaded by two equal and opposite bending moments M, it may be stated that its cross section alters its circular shape into an oval one, owing to the fact, that, apart from the normal bending stresses in the cross section of the tube, there arise tangential bending-stresses in its meridional sections. A closer examination of the fact learns, that the curvature $\frac{1}{\varrho}$ of the "axis" of the tube does not increase proportionally with the loading moment M.

If "a" be the radius of the tube, h its thickness, E the elasticity-modulus of the material and ν the reciprocal value of POISSON's coefficient, the following relation between M and ϱ exists:

$$M = \pi a^3 h \left[\frac{1}{\varrho} - \frac{3}{2} (1 - \nu^2) \frac{a^4}{\varrho^3 h^2} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the graphical representation of which is shown qualitatively in fig. 1.

It is seen from this figure that there exists a critical value M_{crit} of M, characterized by the fact, that no increase of M occurs, if $1/\varrho$ increases. Hence a break-down of the tube is to be expected.

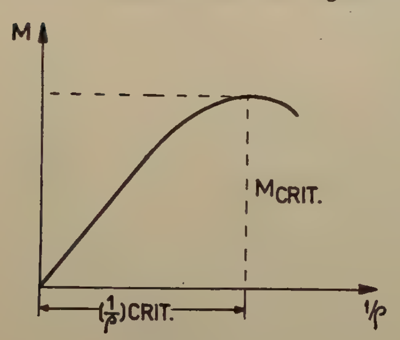

Fig. 1.

The phenomenon here described has been studied at great length by BRAZIER [1]).

In the present paper quite another phenomenon is studied, which occurs for certain critical values of M, and is characterized by the simultaneous appearance of longitudinal and circumferential waves in the cylindrical shape of the tube. We assume, that, — if unloaded —, the tube possesses such initial curvature, that under the action of the buckling moment it is straight, and — in cross-section — circular and of constant thickness h. Hereby our buckling problem relates to

1) Comp. BRAZIER, Aeronautical Research Committee, Reports and Memoranda No. 1081, M 49.

a *straight* circular tube, loaded at his ends by linear changing bending stresses.

Preliminary we shall have to solve some detail-questions (sections 2—6). In section 2 the formulae for the displacements and stresses of a cylindrical tube, submitted to prescribed radial, tangential and axial stresses, R, Φ and Z are reproduced. In section 3 a system of particular loads B is defined and calculated, which plays a fundamental role in our proper buckling problem. The loads B will be called "*elementary normal loads*", the corresponding deformations, "elementary normal deformations". Section 4 deals with the differential equations of the buckling problem; more particularly it is shown that the differential equations for the displacements obtained in section 2 may be looked upon as the required buckling-equations, provided that R, Φ and Z be replaced by adequate "would-be" forces. The so-obtained differential equations are homogeneous and linear in the displacements and therefore only admit solutions (different from zero) for special values of the loading moment $M = \mu \overline{M}$ ($\overline{M}$ = unit of bending moment). The values μ, for which buckling of the tube is possible, will be called the "total characteristic values" of our problem, the corresponding deformations T of the tube "total normal deformations". Section 5 bears on the development of the "total normal deformations" into a series of the "elementary normal deformations". Finally it is shown in section 6, how by iteration the smallest characteristic value μ can approximately be calculated.

2. *The cylindrical circular tube of constant thickness under a prescribed load-system.* As shown in fig. 2 the position of an arbitrary point of

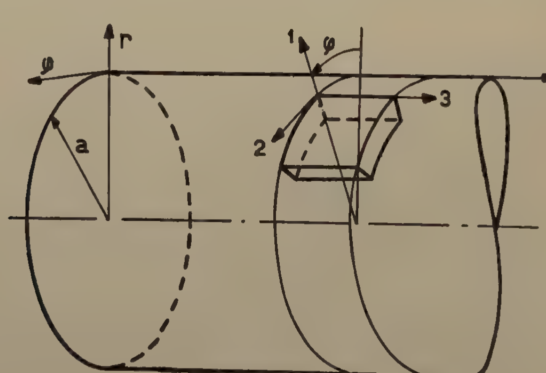

Fig. 2.

the middle-surface of the tube is given by its "cylinder"-coordinates a, φ, and z. The radial, tangential and axial displacements of such a point are called u, v, w; the components of the external load of the tube with reference to the unit of surface, and taken in radial, tangential and

axial direction are designed by R, Φ, and Z. Fig. 3 shows the nomenclature of the so-called "internal" forces and moments, all of which refer to the unit of length of their corresponding section.

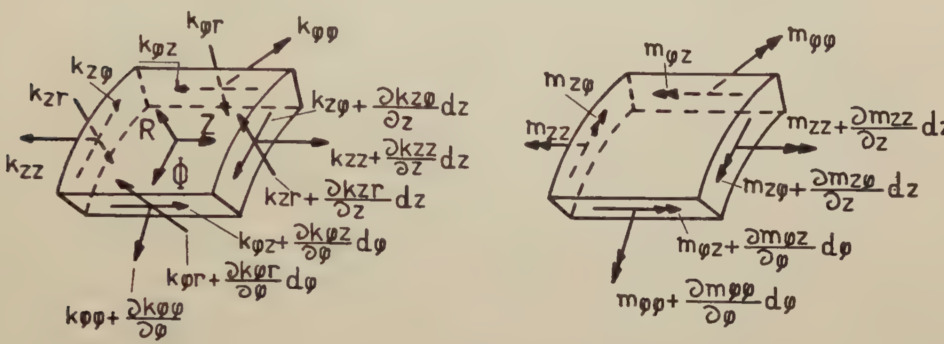

Fig. 3.

If, for abbreviation, we put

$$A^* = \frac{Eh^3}{12(1-\nu^2)}, \quad B = \frac{Eh}{1-\nu^2} \quad \left(\nu = \frac{1}{m}\right) \quad . \quad . \quad . \quad . \quad (1)$$

and for later purposes

$$\frac{A^*}{Ba^2} = \frac{h^2}{12a} = k, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1^*)$$

the equations for u, v, w may be written as follows:

$$\left.\begin{aligned}
&\frac{1}{a}\left(\frac{u}{a} + \frac{1}{a}\frac{\partial v}{\partial \varphi} + \nu\frac{\partial w}{\partial z}\right) + \frac{A^*}{B}\left(\frac{1}{a^4}\frac{\partial^4 u}{\partial \varphi^4} + \frac{2}{a^2}\frac{\partial^4 u}{\partial \varphi^2\,\partial z^2} + \frac{\partial^4 u}{\partial z^4} + \right.\\
&\left. + \frac{2}{a^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{u}{a^4} - \frac{3-\nu}{2a^2}\frac{\partial^3 v}{\partial \varphi\,\partial z^2} + \frac{1-\nu}{2a^3}\frac{\partial^3 w}{\partial \varphi^2\,\partial z} - \frac{1}{a}\frac{\partial^3 w}{\partial z^3}\right) - \frac{R}{B} = 0\\
&\frac{1}{a^2}\frac{\partial u}{\partial \varphi} + \frac{1}{a^2}\frac{\partial^2 v}{\partial \varphi^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial z^2} + \frac{1+\nu}{2a}\frac{\partial^2 w}{\partial \varphi\,\partial z} + \frac{A^*}{B}\left(-\frac{3-\nu}{2a^2}\frac{\partial^3 u}{\partial \varphi\,\partial z^2} + \right.\\
&\left. + 3\,\frac{1-\nu}{2a^2}\frac{\partial^2 v}{\partial z^2}\right) + \frac{\Phi}{B} = 0\\
&\frac{\nu}{a}\frac{\partial u}{\partial z} + \frac{\nu+1}{2a}\frac{\partial^2 v}{\partial \varphi\,\partial z} + \frac{1-\nu}{2a^2}\frac{\partial^2 w}{\partial \varphi^2} + \frac{\partial^2 w}{\partial z^2} + \frac{A^*}{B}\left(\frac{1-\nu}{2a^3}\frac{\partial^3 u}{\partial \varphi^2\,\partial z} - \right.\\
&\left. - \frac{1}{a}\frac{\partial^3 u}{\partial z^3} + \frac{1-\nu}{2a^4}\frac{\partial^2 w}{\partial \varphi^2}\right) + \frac{Z}{B} = 0.
\end{aligned}\right\} . \quad (2)$$

The internal forces and moments can be calculated from u, v, w, by the relations (3):

$$\left.\begin{aligned}
k_{\phi\phi} &= \frac{A^*}{a^3}\left(\frac{\partial^2 u}{\partial\varphi^2}+u\right)+B\left(\frac{u}{a}+\frac{1}{a}\frac{\partial v}{\partial\varphi}+\nu\frac{\partial w}{\partial z}\right)\\
k_{\phi z} &= \frac{(1-\nu)\,A^*}{2a^2}\left(\frac{\partial^2 u}{\partial\varphi\,\partial z}+\frac{1}{a}\frac{\partial w}{\partial\varphi}\right)+\frac{(1-\nu)\,B}{2}\left(\frac{\partial v}{\partial z}+\frac{1}{a}\frac{\partial w}{\partial\varphi}\right)\\
m_{\phi\phi} &= \frac{(1-\nu)\,A^*}{2a}\left(2\frac{\partial^2 u}{\partial\varphi\,\partial z}-\frac{\partial v}{\partial z}+\frac{1}{a}\frac{\partial w}{\partial\varphi}\right)\\
k_{z\phi} &= \frac{(1-\nu)\,A^*}{2a^2}\left(-\frac{\partial^2 u}{\partial\varphi\,\partial z}+\frac{\partial v}{\partial z}\right)+\frac{(1-\nu)\,B}{2}\left(\frac{\partial v}{\partial z}+\frac{1}{a}\frac{\partial w}{\partial\varphi}\right)\\
k_{zz} &= -\frac{A^*}{a}\frac{\partial^2 u}{\partial z^2}+B\left(\nu\frac{u}{a}+\frac{\nu}{a}\frac{\partial v}{\partial\varphi}+\frac{\partial w}{\partial z}\right)\\
m_{z\phi} &= A^*\left(\frac{\nu}{a^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{\partial^2 u}{\partial z^2}-\frac{\nu}{a^2}\frac{\partial v}{\partial\varphi}-\frac{1}{a}\frac{\partial w}{\partial z}\right)\\
m_{zz} &= \frac{(1-\nu)\,A^*}{a}\left(-\frac{\partial^2 u}{\partial\varphi\,\partial z}+\frac{\partial v}{\partial z}\right)
\end{aligned}\right\} . \quad (3)$$

We see at once, that for the special loads

$$\left.\begin{aligned}
R &= a_{pq}\cos p\varphi \sin\lambda\frac{z}{a}\\
\Phi &= b_{pq}\sin p\varphi \sin\lambda\frac{z}{a}\\
Z &= 0
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

— where p and q design arbitrary positive, integer numbers and λ stands for

$$\lambda = \frac{\pi\, q\, a}{l} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

(l being the length of the cylinder) — the equations (2) admit solutions of the following type

$$\left.\begin{aligned}
u &= u_{pq}\cos p\varphi \sin\lambda\frac{z}{a}\\
v &= v_{pq}\sin p\varphi \sin\lambda\frac{z}{a}\\
w &= w_{pq}\cos p\varphi \cos\lambda\frac{z}{a}
\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

u_{pq}, v_{pq} and w_{pq} representing functions of p and q, which are to be calculated from the equations (7)

$$\left.\begin{aligned}
&[1+k(p^4+2p^2\lambda^2+\lambda^4-2p^2+1)]\,u_{pq}+\left[p+\frac{3-\nu}{2}kp\lambda^2\right]v_{pq}+\\
&\quad+\left[-\nu\lambda+k\left(\frac{1-\nu}{2}p^2\lambda-\lambda^3\right)\right]w_{pq}=\frac{a^2}{B}a_{pq}\\
&\left[p+\frac{3-\nu}{2}kp\lambda^2\right]u_{pq}+\left[p^2+\frac{1-\nu}{2}(1+3k)\lambda^2\right]v_{pq}-\\
&\quad-\frac{1+\nu}{2}p\lambda w_{pq}=\frac{a^2}{B}b_{pq}\\
&\left[-\nu\lambda+k\left(\frac{1-\nu}{2}p^2\lambda-\lambda^3\right)\right]u_{pq}-\frac{1+\nu}{2}p\lambda v_{pq}+\\
&\quad+\left[\lambda^2+\frac{1-\nu}{2}(1+k)p^2\right]w_{pq}=0
\end{aligned}\right\} . \qquad (7)$$

(It may be noted, that for all solutions (7), the displacements u and v are zero for $z=0$ and $z=l$, whereas w is different from zero at the ends of the cylinder). We restrict ourselves to solving u_{pq} and v_{pq} and find, under the essential condition, that the thickness h of the tube be small enough to neglect all terms which contain the parameter k (comp. 1*) in higher than the first degree

$$\left.\begin{aligned}
u_{pq}&=\alpha_{pq}a_{pq}+\beta_{pq}b_{pq}\\
v_{pq}&=\beta_{pq}a_{pq}+\gamma_{pq}b_{pq}
\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (8)$$

α_{pq}, β_{pq} and γ_{pq} standing for:

$$\alpha_{pq}=\left(\frac{T_1}{N}\right)_{pq}\frac{a^2}{B};\ \beta_{pq}\equiv-\left(\frac{T_2}{N}\right)_{pq}\frac{a^2}{B};\ \gamma_{pq}=\left(\frac{T_3}{N}\right)_{pq}\frac{a^2}{B} \quad . \quad . \quad (9)$$

T_1, T_2, T_3, N themselves standing for:

$$\left.\begin{aligned}
&T_1=(1+k)p^4+[2\lambda^2+2(1-\nu)\lambda^2k]p^2+(1+3k)\lambda^4\\
&T_2=p[[1+k(2\lambda^2+1)]p^2+(\nu+2)\lambda^2+2k\lambda^4]\\
&T_3=kp^2(p^2-1)^2+k\frac{2(2-\nu)}{1-\nu}\lambda^2p^4+\left[1+\frac{5-\nu}{1-\nu}k\lambda^4+\right.\\
&\quad\left.+\frac{2(\nu-\nu^2-2)}{1-\nu}k\lambda^2+k\right]p^2+2(1+\nu)\lambda^2+\frac{2}{1-\nu}k(\lambda^6-2\nu\lambda^4)\\
&N=kp^8+k[4\lambda^2-2]p^6+k[6\lambda^4-2(4-\nu)\lambda^2+1]p^4+k[4\lambda^6-\\
&\quad-6\lambda^4+2(2-\nu)\lambda^2]p^2+(1-\nu^2)\lambda^4+k\lambda^8-2\nu\lambda^6k+(4-3\nu^2)\lambda^4k.
\end{aligned}\right\} . \qquad (10)$$

3. *The "elementary normal loads" B, and the corresponding "elementary normal deformations" D.* As "elementary normal loads" B and

corresponding "elementary normal deformations" D we define such loads R_{pq}, Φ_{pq} (comp. 2, 4) and such corresponding displacements u_{pq}, v_{pq}, for which

$$\frac{R_{pq}}{u_{pq}} = \frac{\Phi_{pq}}{v_{pq}} = \omega \,. \quad \ldots \ldots \ldots \quad (1)$$

Condition (1) is identical with the requirement, that the equations

$$\left.\begin{array}{l} a_{pq} = \omega\,(\alpha_{pq}\, a_{pq} + \beta_{pq}\, b_{pq}) \\ b_{pq} = \omega\,(\beta_{pq}\, a_{pq} + \gamma_{pq}\, b_{pq}) \end{array}\right\} \text{ or } \left.\begin{array}{l} (\omega\,\alpha_{pq} - 1)\, a_{pq} + \omega\,\beta_{pq}\, b_{pq} = 0 \\ \omega\,\beta_{pq}\, a_{pq} + (\omega\,\gamma_{pq} - 1)\, b_{pq} = 0 \end{array}\right\} . \quad (2)$$

admit solutions a_{pq}, b_{pq} different from zero. (Comp. 2, 8 and 2, 4).

Therefore the equation

$$\begin{vmatrix} \omega\,\alpha_{pq} - 1 & \omega\,\beta_{pq} \\ \omega\,\beta_{pq} & \omega\,\gamma_{pq} - 1 \end{vmatrix} = 0 \,. \quad \ldots \ldots \ldots \quad (3)$$

has to be satisfied by ω. The two (real) roots of this equation are:

$$\begin{array}{c} 1/\omega^{*}_{pq} \\ \text{resp.} \\ 1/\omega^{**}_{pq} \end{array} = \frac{\alpha_{pq} + \gamma_{pq}}{2} \pm \sqrt{\left(\frac{\alpha_{pq} - \gamma_{pq}}{2}\right)^2 + \beta_{pq}{}^2} \quad \ldots \ldots \quad (4)$$

The corresponding loads and displacements — which of course are definite except for a factor of proportionality $\varkappa$ — may readily be calculated from the equations (3).

With:

$$\left.\begin{array}{ll} E^{*}_{pq} \equiv -\dfrac{\alpha_{pq} - \gamma_{pq}}{2\beta_{pq}} + \sqrt{\left(\dfrac{\alpha_{pq} - \gamma_{pq}}{2\beta_{pq}}\right)^2 + 1}\,; & E^{**}_{pq} \equiv -\dfrac{\alpha_{pq} - \gamma_{pq}}{2\beta_{pq}} - \sqrt{\left(\dfrac{\alpha_{pq} - \gamma_{pq}}{2\beta_{pq}}\right)^2 + 1} \\ F^{*}_{pq} \equiv \dfrac{\alpha_{pq} + \gamma_{pq}}{2} + \sqrt{\left(\dfrac{\alpha_{pq} - \gamma_{pq}}{2}\right)^2 + \beta^2_{pq}}\,; & F^{**}_{pq} \equiv \dfrac{\alpha_{pq} + \gamma_{pq}}{2} - \sqrt{\left(\dfrac{\alpha_{pq} - \gamma_{pq}}{2}\right)^2 + \beta^2_{pq}} \\ G^{*}_{pq} = E^{*}_{pq}\, F^{*}_{pq}\,; & G^{**}_{pq} = E^{**}_{pq}\, F^{**}_{pq} \end{array}\right\} (5)$$

they can be written as follows:

$$\left.\begin{array}{ll} R^{*} = \varkappa^{*} \cos p\varphi \sin \lambda \dfrac{z}{a} & R^{**} = \varkappa^{**} \cos p\varphi \sin \lambda \dfrac{z}{a} \\ \Phi^{*} = \varkappa^{*}\, E^{*}_{pq} \sin p\varphi \sin \lambda \dfrac{z}{a} & \Phi^{**} = \varkappa^{**}\, E^{**}_{pq} \sin p\varphi \sin \lambda \dfrac{z}{a} \\ u^{*} = \varkappa^{*}\, F^{*}_{pq} \cos p\varphi \sin \lambda \dfrac{z}{a} & u^{**} = \varkappa^{**}\, F^{**}_{pq} \cos p\varphi \sin \lambda \dfrac{z}{a} \\ v^{*} = \varkappa^{*}\, G^{*}_{pq} \sin p\varphi \sin \lambda \dfrac{z}{a} & v^{**} = \varkappa^{**}\, G^{**}_{pq} \sin p\varphi \sin \lambda \dfrac{z}{a} \end{array}\right\} . \quad (6)$$

the first solution obviously belonging to ω^*, the second one to ω^{**}. To get rid of the undeterminate factors $\varkappa^*$ and $\varkappa^{**}$ we standardize our solutions by the condition of standardization

$$\int_0^l \int_0^{2\pi} (u^* R^* + v^* \Phi^*)\, a d\varphi\, dz = \frac{\pi\, al}{2} \text{ resp. } \int_0^l \int_0^{2\pi} (u^{**} R^{**} + v^{**} \Phi^{**})\, a d\varphi\, dz = \frac{\pi\, al}{2} \quad (7)$$

and find hereby:

$$\varkappa^* = \frac{1}{\sqrt{F^*_{pq} + E^*_{pq}\, G^*_{pq}}} \text{ resp. } \varkappa^{**} = \frac{1}{\sqrt{F^{**}_{pq} + E^{**}_{pq}\, G^{**}_{pq}}} \quad . \quad . \quad . \quad (8)$$

Henceforth the expression "elementary normal function" will be used in the same sense as "standardized elementary normal function", so that from now the elementary normal loads and deformations will be represented by the eqs. (6) and (8).

In the following sections we shall have to deal with systems of elementary normal functions, belonging to a fixed value of the second affix q. In such case this affix will be suppressed. It can easily be proved, that, under this understanding, the following so-called "orthogonality"-relations exist between the functions (6):

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\displaystyle\int_0^l \int_0^{2\pi} [R^*_k u^*_l + \Phi^*_k v^*_l]\, a d\varphi dz = 0 \\
\displaystyle\int_0^l \int_0^{2\pi} [R^{**}_k u^{**}_l + \Phi^{**}_k v^{**}_l]\, a d\varphi dz = 0
\end{array}\right\} k \neq l \\
\left.\begin{array}{l}
\displaystyle\int_0^l \int_0^{2\pi} [R^*_k u^{**}_l + \Phi^*_k v^{**}_l]\, a d\varphi dz = 0 \\
\displaystyle\int_0^l \int_0^{2\pi} [R^{**}_k u^*_l + \Phi^{**}_k v^*_l]\, a d\varphi dz = 0
\end{array}\right\} \begin{array}{l}\text{for all}\\ \text{values}\\ \text{of } k\\ \text{and } l.\end{array}
\end{array}\right\} \begin{array}{l} k \text{ and } l \text{ denoting} \\ \text{values of the} \\ \text{first affix } p; \\ \text{the second affix} \\ q \text{ being fixed.} \end{array} \Bigg\} \quad . \quad . \quad (9)$$

4. *The differential-equations of the buckling tube.* If only the load components R, Φ and Z and the displacements u, v, w be adequately interpreted, the equations (2, 2) may be looked upon as the differential-equations of the buckling tube, loaded by axial (and linearly changing) bending stresses at its endsections. As stated in section 1, we assume that, — thanks to its initial curvature — the tube, under influence of its buckling moments $M = \mu \overline{M}$, can be regarded as straight and of constant thickness. The displacements u, v, w of any point of the middle-surface, *from this initial state of stress and strain* (II) replace from this moment the quantities, designed in the same way in equation 2, 2, denoting originally the displacements from the *unstressed state* (I).

One could be inclined, in applying the equations (2, 2) to the buckling problem, to put R, Φ and Z equal to zero, due to the fact that these quantities evidently now stand for the *supplementary* loads, which eventually arise with the transition from state II to the (indefinitely) neighbouring buckling-configuration (II′). The following fact, however, has to be observed. An arbitrary element of the tube, as represented in fig. 4, being in state II, finds itself in equilibrium, under the axial forces $Q\,ad\varphi$, Q representing the external force pro unit of circumferential length exerted on the end-sections of the tube in the points $(z=0, \varphi)$ and $(z=l, \varphi)$.

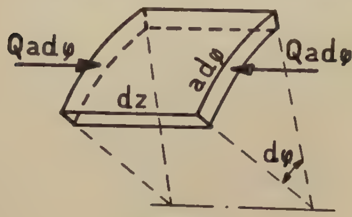

Fig. 4.

If now the transition takes place from state II to state II′ these forces retain their magnitude (because the external load of the tube does not alter at all), but they change in direction in accordance with the change in shape and curvature of the surface-elements on which they act. Therefore these forces (though unchanged in magnitude) produce components in the directions r, φ and z, which, divided by the surface $ad\varphi\,dz$, play the role of R, Φ and Z in the equations (2, 2).

It can be shown [1]), that in our case R, Φ, Z amount to

$$R=-Q\frac{\partial^2 u}{\partial z^2},\quad \Phi=-Q\frac{\partial^2 v}{\partial z^2},\quad Z=0 \quad . \quad . \quad . \quad . \quad (1)$$

and therefore the differential-equations of our buckling problem run as follows:

$$\left.\begin{aligned}
&\frac{1}{a}\left(\frac{u}{a}+\frac{1}{a}\frac{\partial v}{\partial\varphi}+\nu\frac{\partial w}{\partial z}\right)+\frac{A^*}{B}\left(\frac{1}{\alpha^4}\frac{\partial^4 u}{\partial\varphi^4}+\frac{2}{a^2}\frac{\partial^4 u}{\partial\varphi^2\partial z^2}+\frac{\partial^4 u}{\partial z^4}+\right.\\
&\qquad\left.+\frac{2}{a^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{u}{a^4}-\frac{3-\nu}{2a^2}\frac{\partial^3 v}{\partial\varphi\,\partial z^2}+\frac{1-\nu}{2a^3}\frac{\partial^3 w}{\partial\varphi^2\partial z}-\frac{1}{a}\frac{\partial^3 w}{\partial z^3}\right)+\\
&\qquad+\frac{Q}{B}\frac{\partial^2 u}{\partial z^2}=0\\
&\frac{1}{a^2}\frac{\partial u}{\partial\varphi}+\frac{1}{a^2}\frac{\partial^2 v}{\partial\varphi^2}+\frac{1-\nu}{2}\frac{\partial^2 v}{\partial z^2}+\frac{1+\nu}{2a}\frac{\partial^2 w}{\partial\varphi\,\partial z}+\frac{A^*}{B}\left(-\frac{3-\nu}{2a^2}\frac{\partial^3 u}{\partial\varphi\partial z^2}+\right.\\
&\qquad\left.+3\,\frac{1-\nu}{2a^2}\frac{\partial^2 v}{6 z^2}\right)-\frac{Q}{B}\frac{\partial^2 v}{\partial z^2}=0\\
&\frac{\nu}{a}\frac{\partial u}{\partial z}+\frac{1+\nu}{2a}\frac{\partial^2 v}{\partial\varphi\,\partial z}+\frac{1-\nu}{2a^2}\frac{\partial^2 w}{\partial\varphi^2}+\frac{\partial^2 w}{\partial z^2}+\frac{A^*}{B}\left(\frac{1-\nu}{2a^3}\frac{\partial^3 u}{\partial\varphi^2\partial z}-\right.\\
&\qquad\left.-\frac{1}{a}\frac{\partial^3 u}{\partial z^3}\right)+\frac{1-\nu}{2a^4}\frac{\partial^2 w}{\partial\varphi^2}=0
\end{aligned}\right\}\quad . \quad (2)$$

[1]) Comp. W. FLÜGGE, Statik und Dynamik der Schalen: Springer, Berlin 1934. C. B. BIEZENO and R. GRAMMEL, Technische Dynamik; Springer, Berlin 1939.

There only remains to express $\frac{Q}{B}$ in terms of the loading moment $M = \mu \overline{M}$. We find (comp. fig. 5 and 2, 1):

$$\frac{Q}{B} = \frac{M a \cos\varphi}{\pi a^3 h} h . \frac{(1-\nu^2)}{E h} = \frac{\mu \overline{M} (1-\nu^2)}{\pi a^2 E h} \cos\varphi \quad . \quad . \quad . \quad . \quad (3)$$

It has already been stated in section 1, that the eqs. (2) are homogeneous in u, v, w and their derivatives, and therefore only have solutions, different from $u = v = w = 0$, for distinct values of the parameter μ, respectively for distinct values of the loading moment $M = \mu \overline{M}$. Those values will be called the "total characteristic numbers" respectively the "total characteristic moments" of our problem; the corresponding deformations (T) the "total characteristic deformations". From now it is our subject to calculate the *smallest* characteristic moment.

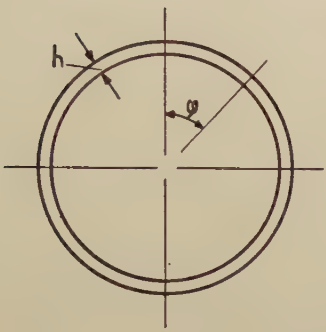

Fig. 5.

5. *The expansion of a "total characteristic deformation" T into a series of elementary normal deformations D. The influence numbers a_{ij}.* The main result of the preceding section lays in the fact, that in case of buckling the external loading moments M of the tube give rise to a "would-be" surface load R, Φ, Z given by

$$R = -Q \frac{\partial^2 u}{\partial z^2} = -\frac{M}{\pi a^2} \frac{\partial^2 u}{\partial z^2} \cos\varphi, \quad \Phi = -Q \frac{\partial^2 v}{\partial z^2} = -\frac{M}{\pi a^2} \frac{\partial^2 v}{\partial z^2} \cos\varphi, \quad Z = 0 \quad (1)$$

the magnitude of which depends upon the total characteristic deformation u, v, w, that corresponds with M. We learn from (1) that — if this total deformation be decomposed in a set of other deformations $u_1, v_1, w_1; u_2, v_2, w_2 \ldots$ —, such that $u = u_1 + u_2 + \ldots$, $v = v_1 + v_2 + \ldots$, $w = w_1 + w_2 + \ldots$, the loadsystem (1) may be decomposed in a set of other loadsystems, each of which is calculable by (1) from u_1, v_1, w_1, u_2, v_2, w_2 etc.

If therefore the total characteristic deformation T be expanded in a (infinite) series of "elementary normal deformations" D:

$$T = \sum_{i=1}^{\infty} d_i D_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad , \quad (2)$$

and if the loadsystem, derived from D_i with the aid of (1) be called $\overline{B}_i$ then the "would-be" loadsystem belonging to the total characteristic deformation under consideration, may be written as:

$$\Sigma \, d_i \overline{B}_i .$$

Each system $\overline{B}_i$, on its part, can be expanded in a (finite or infinite)

series of elementary normal loadsystems B, so that, at the end the would-be loadsystem (1), produced by the total characteristic deformation under consideration, can be regarded upon as the sum of an infinite number of groups of a (finite or infinite) number of elementary normal loads B.

Two remarks of some importance are here to be made. Firstly it has been silently assumed, that all deformations and load systems, introduced in this section belong to the *same* parameter λ (comp. (2, 5)), which is characteristic for the total deformation T, so that all deformations and loads, considered here, have the same number of longitudinal waves. In accordance with a remark, already made in section 3, we therefore denote the functions T, B and D by one single suffix p, relating to the number of circumferential waves.

Secondly, we could — of course — have developed *directly* the load system (1) into a series of the elementary normal loads. We did not, however, proceed in this way in view of the introduction of a system of so-called influence-numbers a_{ij}, which will now be defined.

If (artificially) the tube be given the elementary normal deformation D, then we can — by the aid of (1) — formally calculate the "would-be" loadsystems roused by two unit bending moments $\overline{M}$, acting at the ends of the tube. As stated before, this loadsystem can be developed in a (finite or infinite) series of elementary, normal functions B. The coefficient a_{ij}, which in this expansion belongs to the elementary normal function B_i is called "the influence number *of* the elementary normal deformation D_j *with respect to* the elementary normal load B_i.

This formal definition provides us with an expedient to obtain system of homogeneous linear equations for the coefficients d_i in the expression (2). Indeed, if a deformation D_j provokes a "would-be" load, which contains a_{ij} times B_i, (assumed that the tube be charged by unit bending moments $\overline{M}$), then a deformation $d_j D_j$ provokes a would-be load, which contains $\mu d_j a_{ij}$ times B_i, assumed that the tube be charged by two bending moments $M = \mu \overline{M}$.

The total characteristic deformation $T = \sum\limits_{i=1}^{\infty} d_i D_i$ therefore provokes a would-be load, which contains B_i

$$\sum_{j=1}^{\infty} \mu\, d_j\, a_{ij} \text{ times.}$$

On the other hand it was stated in section 2, that the deformation $T = \sum\limits_{i=1}^{\infty} d_i D_i$ only can be maintained by the load $\sum\limits_{i=1}^{\infty} d_i B_i$, and therefore the system of equations

$$d_i = \sum_{j=1} \mu\, d_j\, a_{ij} \quad i = 1, 2 \ldots \qquad (3)$$

must hold.

It only has solutions different from $d_i = 0$ $(i = 1, 2 \ldots)$ if $\frac{1}{\mu}$ satisfies the equation:

$$\begin{vmatrix} a_{11} - \frac{1}{\mu} & a_{12} & a_{13} \ldots\ldots \\ a_{21} & a_{22} - \frac{1}{\mu} & a_{23} \ldots\ldots \\ a_{31} & a_{32} & a_{33} - \frac{1}{\mu} \ldots \\ \vdots & \vdots & \vdots \quad \ddots \end{vmatrix} = 0 \quad \ldots \quad (4)$$

which formally can be considered as representing our problem.

Now we proceed by proving, that — if only the elementary normal functions be suitably numbered — the reciprocal relation $a_{ij} = a_{ji}$ holds, so that equation (4) is a secular one, possessing only real roots. To this end we calculate by using (1) the would-be loads R, Φ — to be denoted by $\overline{R}^*$, $\overline{\Phi}^*$, resp. $\overline{R}^{**}$, $\overline{\Phi}^{**}$ — belonging to the elementary normal deformations u^*, v^*, resp. u^{**}, v^{**} defined by (3, 6) and (3, 8). They are — if the "order" of the underlying elementary deformation be indicated by p —

$$\left.\begin{aligned} \overline{R}^*_p &= \frac{\lambda^2 F^*_p}{2\pi a^2 \sqrt{F^*_p + E^*_p G^*_p}} [\cos(p+1)\varphi + \cos(p-1)\varphi] \sin \frac{\lambda z}{a} \\ \overline{\Phi}^*_p &= \frac{\lambda^2 G^*_p}{2\pi a^2 \sqrt{F^*_p + E^*_p G^*_p}} [\sin(p+1)\varphi + \sin(p-1)\varphi] \sin \frac{\lambda z}{a} \\ \overline{R}^{**}_p &= \frac{\lambda^2 F^{**}_p}{2\pi a^2 \sqrt{F^{**}_p + E^{**}_p G^{**}_p}} [\cos(p+1)\varphi + \cos(p-1)\varphi] \sin \frac{\lambda z}{a} \\ \overline{\Phi}^{**}_p &= \frac{\lambda^2 G^{**}_p}{2\pi a^2 \sqrt{F^{**}_p + E^{**}_p G^{**}_p}} [\sin(p+1)\varphi + \sin(p-1)\varphi] \sin \frac{\lambda z}{a} \end{aligned}\right\} \quad . \quad . \quad (5)$$

Both load-systems $\overline{R}^*_p$, $\overline{\Phi}^*_p$, and $\overline{R}^{**}_p$, $\overline{\Phi}^{**}_p$ can be linearly expressed in the elementary load-systems

$$B^*_{p-1} = (R^*_{p-1}, \Phi^*_{p-1}); \quad B^*_{p+1} = (R^*_{p+1}, \Phi^*_{p+1}); \quad B^{**}_{p-1} = (R^{**}_{p-1}, \Phi^{**}_{p-1});$$
$$B^{**}_{p+1} = (R^{**}_{p+1}, \Phi^{**}_{p+1}).$$

If we restrict ourselves to the system $\overline{R}^*_p$, $\overline{\Phi}^*_p$, and if we put

$$\left.\begin{aligned}\overline{R}^*_p &= \delta^1_p\, R^*_{p-1} + \delta^2_p\, R^{**}_{p-1} + \delta^3_p\, R^*_{p+1} + \delta^4_p\, R^{**}_{p+1} \\ \overline{\Phi}^*_p &= \delta^1_p\, \Phi^*_{p-1} + \delta^2_p\, \Phi^{**}_{p-1} + \delta^3_p\, \Phi^*_{p+1} + \delta^4_p\, \Phi^{**}_{p+1}\end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

the coefficients δ^i_p $(i = 1, 2 \ldots 4)$ can be found by using the relations

$$\left.\begin{aligned}\frac{\pi a l}{2}\,\delta^1_p &= \int_0^l \int_0^{2\pi} [\overline{R}^*_p\, \overset{*}{u}_{p-1} + \overline{\Phi}^*_p\, \overset{*}{v}_{p-1}]\, a\, d\varphi\, dz \\ \frac{\pi a l}{2}\,\delta^2_p &= \int_0^l \int_0^{2\pi} [\overline{R}^*_p\, \overset{**}{u}_{p-1} + \overline{\Phi}^*_p\, \overset{**}{v}_{p-1}]\, a\, d\varphi\, dz \\ \frac{\pi a l}{2}\,\delta^3_p &= \int_0^l \int_0^{2\pi} [\overline{R}^*_p\, \overset{*}{u}_{p+1} + \overline{\Phi}^*_p\, \overset{*}{v}_{p+1}]\, a\, d\varphi\, dz \\ \frac{\pi a l}{2}\,\delta^4_p &= \int_0^l \int_0^{2\pi} [\overline{R}^*_p\, \overset{**}{u}_{p+1} + \overline{\Phi}^*_p\, \overset{**}{v}_{p+1}]\, a\, d\varphi\, dz\end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

which can be verified by substituting the expressions (6) in the right-hand members and by taking into account the relations of orthogonality established in section 3 (comp. 3, 9). Substitution of the explicite (5) of $\overline{R}^*_p$, $\overline{\Phi}^*_p$, $\overline{R}^{**}_p$, $\overline{\Phi}^{**}_p$ into (7) gives:

$$\left.\begin{aligned}\delta^1_p &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F^*_p\, F^*_{p-1} + G^*_p\, G^*_{p-1}}{\sqrt{F^*_p + E^*_p\, G^*_p}\,\sqrt{F^*_{p-1} + E^*_{p-1}\, G^*_{p-1}}}\right] \\ \delta^2_p &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F^*_p\, F^{**}_{p-1} + G^*_p\, G^{**}_{p-1}}{\sqrt{F^*_p + E^*_p\, G^*_p}\,\sqrt{F^{**}_{p-1} + E^{**}_{p-1}\, G^{**}_{p-1}}}\right] \\ \delta^3_p &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F^*_p\, F^*_{p+1} + G^*_p\, G^*_{p+1}}{\sqrt{F^*_p + E^*_p\, G^*_p}\,\sqrt{F^*_{p+1} + E^*_{p+1}\, G^*_{p+1}}}\right] \\ \delta^4_p &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F_p\, F_{p+1} + G_p\, G_{p+1}}{\sqrt{F^*_p + E^*_p\, G^*_p}\,\sqrt{F^{**}_{p+1} + E^{**}_{p+1}\, G^{**}_{p+1}}}\right]\end{aligned}\right\} \quad . \quad . \quad (8)$$

If again we put

$$\left.\begin{aligned}\overline{R}^{**}_p &= \varepsilon^1_p\, R^*_{p-1} + \varepsilon^2_p\, R^{**}_{p-1} + \varepsilon^3_p\, R^*_{p+1} + \varepsilon^4_p\, R^{**}_{p+1} \\ \overline{\Phi}^{**}_p &= \varepsilon^1_p\, \Phi^*_{p-1} + \varepsilon^2_p\, \Phi^{**}_{p-1} + \varepsilon^3_p\, \Phi^*_{p+1} + \varepsilon^4_p\, \Phi^{**}_{p+1}\end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

we find in an analogous way

$$\left.\begin{aligned}
\varepsilon_p^1 &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F_p^{**} F_{p-1}^{*} + G_p^{**} G_{p-1}^{*}}{\sqrt{F_p^{**} + E_p^{**} G_p^{**}}\sqrt{F_{p-1}^{*} + E_{p-1}^{*} G_{p-1}^{*}}}\right] \\
\varepsilon_p^2 &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F_p^{**} F_{p-1}^{**} + G_p^{**} G_{p-1}^{**}}{\sqrt{F_p^{**} + E_p^{**} G_p^{**}}\sqrt{F_{p-1}^{**} + E_{p-1}^{**} G_{p-1}^{**}}}\right] \\
\varepsilon_p^3 &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F_p^{**} F_{p+1}^{*} + G_p^{**} G_{p+1}^{*}}{\sqrt{F_p^{**} + E_p^{**} G_p^{**}}\sqrt{F_{p+1}^{*} + E_{p+1}^{*} G_{p+1}^{*}}}\right] \\
\varepsilon_p^4 &= \frac{\lambda^2}{2\pi a^2}\left[\frac{F_p^{**} F_{p+1}^{**} + G_p^{**} G_{p+1}^{**}}{\sqrt{F_p^{**} + E_p^{**} G_p^{**}}\sqrt{F_{p+1}^{**} + E_{p+1}^{**} G_{p+1}^{**}}}\right]
\end{aligned}\right\} \quad . \quad . \quad (10)$$

It goes without saying that each of the coefficients δ_p^i and ε_p^i $(i=1,2\ldots4)$ represents an influence-number α_{ij} in the previously defined sense. Furthermore it is evident, that no other influence-coefficients exist except those represented by (8) and (10).

The question, which suffixes must be ascribed to the coefficient α, to let it represent a given δ or ε, depends upon the way in which the elementary normal deformations $D_i \equiv (\overset{*}{u}_i, \overset{*}{v}_i)$ and $D_i^{**} \equiv (\overset{**}{u}_i, \overset{**}{v}_i)$ are arranged. We fix, that

$D_0^* \equiv 0$ will be indicated by D_i $(i=0)$ and in consequence B_0^* by B_i $(i=0)$
D_0^{**} „ „ „ „ D_i $(i=1)$ „ „ „ B_0^{**} by B_i $(i=1)$
D_1^{*} „ „ „ „ D_i $(i=2)$ „ „ „ B_1^{*} by B_i $(i=2)$
D_1^{**} „ „ „ „ D_i $(i=3)$ „ „ „ B_1^{**} by B_i $(i=3)$
a. s. o.

Bearing in mind that δ_p^1 represents the influence-coefficient of the elementary normal load B_{p-1}^* with respect to the elementary normal deformation D_p^*, and that B_{p-1}^* and D_p^* in the just defined sequences of normal loads B_i and normal deformations D_i have the numbers $i=2(p-1)$, resp. $i=2p$, then it is obvious that δ_p^1 has to be called $\alpha_{2(p-1),2p}$, and that the significance of the coefficients δ_p^i, ε_p^i in general can be derived from the following scheme

$$\left.\begin{aligned}
\delta_p^1 &\equiv \alpha_{2(p-1),2p} & \varepsilon_p^1 &= \alpha_{2(p-1),2p+1} \\
\delta_p^2 &\equiv \alpha_{2(p-1)+1,2p} & \varepsilon_p^2 &= \alpha_{2(p-1)+1,2p+1} \\
\delta_p^3 &\equiv \alpha_{2(p+1),2p} & \varepsilon_p^3 &= \alpha_{2(p+1),2p+1} \\
\delta_p^4 &\equiv \alpha_{2(p+1)+1,2p} & \varepsilon_p^4 &= \alpha_{2(p+1)+1,2p+1}
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From the equations (8) and (10) we deduce

$$\delta_p^1 = \delta_{p-1}^3 \ , \ \varepsilon_p^1 = \delta_{p-1}^4 \ , \ \delta_p^2 = \varepsilon_{p-1}^3 \ , \ \varepsilon_p^2 = \varepsilon_{p-1}^4 \, ,$$

and consequently from equations (11)

$$\left.\begin{aligned} \alpha_{2(p-1),2p} &= \alpha_{2p,2(p-1)} \\ \alpha_{2(p-1),2p+1} &= \alpha_{2p+1,2(p-1)} \\ \alpha_{2(p-1)+1,2p} &= \alpha_{2p,2(p-1)+1} \\ \alpha_{2(p-1)+1,2p+1} &= \alpha_{2p+1,2(p-1)+1} \end{aligned}\right\} \quad \dots\dots\dots \quad (12)$$

These equations obviously can be contracted in the single relation of reciprocity:

$$a_{ij} = a_{ji}. \quad \dots\dots\dots\dots \quad (13)$$

The numbering of normal functions, introduced in this section, therefore make the left-hand side of (4) to a symmetrical determinant. If furthermore we give attention to the fact, that in our new nomenclature influence-coefficients of the type a_{ii} do not occur, and that a great number of the other coefficients is zero, equation (4) can be replaced by:

$$\begin{vmatrix} \frac{1}{\mu} & a_{12} & a_{13} & 0 & 0 & 0 & 0 \dots \\ a_{21} & -\frac{1}{\mu} & 0 & a_{24} & a_{25} & 0 & 0 \dots \\ a_{31} & 0 & -\frac{1}{\mu} & a_{34} & a_{35} & 0 & 0 \dots \\ 0 & a_{42} & a_{43} & -\frac{1}{\mu} & 0 & a_{46} & a_{47} \dots \\ 0 & a_{52} & a_{53} & 0 & -\frac{1}{\mu} & a_{56} & a_{57} \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{vmatrix} = 0 \quad (14)$$

with $a_{ij} = a_{ji}$.

After a well-known theorem the roots of this equation are one and all real. If the columns 2, 3, 6, 7, 10, 11,... and the rows 1, 4, 5, 8, 9,... of the determinant are multiplied by -1, no alteration takes place in its general shape, except that all terms $\frac{1}{\mu}$ change their sign. Therefore it can be stated beforehand, that all roots of equation (14) occur in pairs of equal magnitude and opposite sign.

Every root μ_k of (13) corresponds to a total characteristic deformation T_k

$$T_k = \sum_{i=1}^{\infty} d_{ki} D_i,$$

the coefficients d_i satisfying the equations:

$$d_{ki} = \sum_{j=1}^{n} \mu_k a_{ij} d_{kj} \ (i = 1, 2 \dots). \quad \dots\dots \quad (15)$$

Mathematics. — *Zur projektiven Differentialgeometrie der Regelflächen im R_4.* (Vierte Mitteilung). Von R. WEITZENBÖCK.

(Communicated at the meeting of June 29, 1940.)

Ich ermittle in dieser Mitteilung die einfachsten Differentialinvarianten einer Regelfläche F im R_4. Neben der Invariante Q mit den Gewichten 9 und 5 stellen sich zwei Invarianten A und B vom φ'-Gewichte 20 und vom λ-Gewicht 10 ein. Eine lineare Kombination derselben, $U = -36A + 24B$ ergibt sich durch geometrische Betrachtungen.

§ 12.

Die Ausdrücke

$$J_\varrho = h_{ik,rs} = (h\, i^2 k^2)\,(h\, r^2 s^2) = (a_h\, a_i{}^2\, a_k{}^2)\,(a_h\, a_r{}^2\, a_s{}^2)$$

sind vom φ'-Gewichte

$$\varrho = h + i + k + r + s,$$

d.h. bei den Transformationen (7) hat man

$$\widetilde{J}_\varrho = \widetilde{h}_{\widetilde{ik}, \widetilde{rs}} = J_\varrho \cdot (\varphi')^\varrho + J' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

wo J' weitere Glieder mit $\varphi', \varphi'', \ldots$ andeutet. Bei einer *Invariante* J_ϱ ist $J' = 0$.

Da $0_{12, rs}$ und $0_{13, 22}$ verschwinden, müssen wir $\varrho \geqq 9$ haben. Man sieht leicht, dass sich bei $\varrho = 9$ nur eine einzige Invariante J_9 ergibt, nämlich

$$Q = 0_{13, 22}$$

(Vgl. die auf Gleichung (41) folgenden Gleichungen).

Bei $\varrho = 10$ haben wir zunächst mit $h = 0$ die folgenden Ausdrücke J_{10}, die nicht unmittelbar nach den Gleichungen (1) reduzierbar sind:

$$0_{13, 33} = R \quad , \quad 0_{13, 24} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

Bei $h = 1$, also in $1_{ik, rs}$ kann man den Fall $1_{0k, rs}$ weglassen, da er wegen

$$1_{0k, rs} = -0_{1k, rs} - k_{01, rs} = -0_{1k, rs}$$

auf $h = 0$ zurückführt. Es bleibt daher bei $\varrho = 10$ mit $h = 1$ nur

$$1_{13, 23} = -\tfrac{1}{2}\, 3_{11, 23} = +\tfrac{1}{2}\, 3_{02, 23} = -\tfrac{1}{2}\, 2_{03, 23}\,,$$

sodass wir also $h > 1$ annehmen können. Bei $h = 2$ ergeben sich die Ausdrücke

$$2_{03,23}\,,\ 2_{03,05}\,.$$

Hier ist der zweite bereits durch Gleichung (82) durch Q' und R ausgedrückt:

$$2_{03,05} = \tfrac{5}{6}\,R - 5\,Q' \quad \ldots\ldots\ldots \quad (102)$$

Beim ersten haben wir vermöge der zyklischen Symmetrie:

$$2_{03,23} = -\tfrac{1}{2}\,3_{03,22} = +\tfrac{1}{4}\,0_{33,22} = -\tfrac{1}{4}\,0_{22,33} = +\tfrac{1}{3}\,0_{13,33} = \tfrac{1}{3}\,R\,. \quad (103)$$

Die Fälle $h > 2$ lassen sich alle auf (102) und (101) reduzieren.

Für $0_{13,24}$ haben wir bereits oben (vgl. die Gleichung vor Gleichung (82)) gefunden

$$0_{13,24} = -\tfrac{5}{6}\,R + Q'. \quad \ldots\ldots\ldots \quad (104)$$

Hiermit sind alle J_{10} auf R und Q' reduziert.

Bei $\varrho = 11$ ergeben sich für $h = 0$ zunächst wieder die Ausdrücke

$$0_{13,34},\ 0_{13,25},\ 0_{23,33} = S,\ 0_{23,14} \quad \ldots\ldots\ldots \quad (105)$$

Wenn wir $R = 0_{13,33}$ differenzieren, so entsteht

$$R' = 1_{13,33} + 0_{23,33} + 0_{14,33} + 2.0_{13,34}\,.$$

Hier ist der erste Term rechts Null wegen

$$1_{13,33} = -\tfrac{1}{2}\,3_{11,33} = 0;$$

der zweite Term ist S und beim dritten Term drücken wir 14 nach (1) aus:

$$0_{14,33} = -2.0_{23,33} = -2\,S.$$

Also kommt

$$0_{13,34} = \tfrac{1}{2}\,(R' + S). \quad \ldots\ldots\ldots \quad (106)$$

Damit ist der erste der Ausdrücke (105) auf R' und S reduziert.

Bei $h = 1$ gibt 1_{0i} wieder Reduktion auf (105) und alle anderen Ansätze können auf $h > 1$ zurückgebracht werden. Da ferners in $h_{ik,rs}$ bei $h > 3$ $i + k \leqq 3$ vorausgesetzt werden darf, sind nur noch die Fälle $h = 2$ und $h = 3$ zu untersuchen. Aber auch von $h = 3$ kann man hier absehen, da sich alle Ansätze $3_{ik,rs}$ auf $h = 2$ reduzieren lassen. Bei $h = 2$ hat man, da $2_{02,rs}$ auf (105) führt, die folgenden Ausdrücke zu untersuchen:

$$\left.\begin{array}{llll} 2_{03,33} & 2_{03,24} & 2_{03,15} & (i+k=3) \\ 2_{04,23} & 2_{04,14} & & (i+k=4) \end{array}\right\} \quad \ldots\ldots \quad (107)$$

Hier wird nun der erste dieser Ausdrücke

$$2_{03,33} = -0_{23,33} = -S; \quad \ldots\ldots\ldots \quad (108)$$

beim zweiten hat man

$$2_{03,24} = -\tfrac{1}{2} 4_{03,22} = \tfrac{1}{2} 0_{34,22} + \tfrac{1}{2} 3_{04,22},$$

oder wegen

$$0_{34,22} = -\tfrac{4}{3} 0_{34,13} = \tfrac{4}{3} 0_{13,34},$$

also nach Gleichung (106)

$$0_{34,22} = \tfrac{2}{3}(R' + S). \quad \ldots\ldots\ldots \quad (109)$$

und wegen

$$3_{04,22} = -\tfrac{4}{3} 3_{04,13} = +\tfrac{2}{3} 1_{04,33} = -\tfrac{2}{3} 0_{14,33} = \tfrac{4}{3} 0_{23,33} = \tfrac{4}{3} S:$$

$$2_{03,24} = \tfrac{1}{3} R' + S. \quad \ldots\ldots\ldots \quad (110)$$

Beim dritten der Ausdrücke (107) wird

$$2_{03,15} = -1_{03,25} - 5_{03,12} = +0_{13,25}, \quad \ldots\ldots \quad (111)$$

also Reduktion auf (105).

Weiters ist

$$2_{04,23} = -\tfrac{1}{2} 3_{04,22} = +\tfrac{2}{3} 3_{04,13} = -\tfrac{1}{3} 1_{04,33} = +\tfrac{1}{3} 0_{14,33} = -\tfrac{2}{3} 0_{23,33} = -\tfrac{2}{3} S, \quad (112)$$

und schliesslich

$$2_{04,14} = -0_{24,14} - 4_{02,14} = 0_{14,24} = -2.0_{23,24}, \quad \ldots\ldots \quad (113)$$

da

$$4_{02,14} = -\tfrac{1}{2} 1_{02,44} = +\tfrac{1}{2} 1_{11,44} = 0$$

wird.

Hiermit sind alle Ausdrücke (107) auf R', S und (105) reduziert.

Differenzieren wir (104), so entsteht

$$-\tfrac{5}{6} R' + Q'' = 1_{13,24} + 0_{23,24} + 0_{14,24} + 0_{13,34} + 0_{13,25}. \quad \ldots \quad (114)$$

Hier wird der erste Term rechts:

$$1_{13,24} = -2_{13,14} - 4_{13,12} = +\tfrac{1}{4} 2_{04,14} - 4_{13,12},$$

also nach (113):

$$1_{13,24} = -\tfrac{1}{2} 0_{23,24} - 4_{13,12}.$$

Für den letzten Ausdruck rechter Hand haben wir:

$$4_{13,12} = -\tfrac{1}{3} 4_{13,03} = +\tfrac{1}{3} 0_{13,34} + \tfrac{1}{3} 3_{13,04} = \tfrac{1}{3} 0_{13,34} - \tfrac{1}{6} 1_{33,04} =$$
$$= \tfrac{1}{3} 0_{13,34} + \tfrac{1}{6} 0_{33,14} = \tfrac{1}{3} 0_{13,34} - \tfrac{1}{3} 0_{33,23},$$

also nach (106):

$$4_{13,12} = \tfrac{1}{6} R' + \tfrac{1}{2} S.$$

Damit wird

$$1_{13,24} = -\tfrac{1}{2}\, 0_{23,24} - \tfrac{1}{6} R' - \tfrac{1}{2} S. \quad . \quad . \quad . \quad . \quad . \quad (115)$$

Beim dritten Term in (114) erhält man

$$0_{14,24} = -2.0_{23,24}\,.$$

Damit kommt schliesslich statt (114):

$$0_{23,24} = \tfrac{2}{3}\, 0_{13,25} + \tfrac{7}{9} R' - \tfrac{2}{3} Q''. \quad . \quad . \quad . \quad . \quad . \quad (116)$$

Damit sind alle Ausdrücke J_{11} reduziert auf S, R', Q'' und $0_{13,25}$, wobei statt $0_{13,25}$ auch $0_{23,24}$ genommen werden kann.

§ 13.

Die Resultate des vorigen § können wir wie folgt formulieren: Jedes J_{10} hat die Gestalt

$$J_{10} = aR + bQ' = \alpha\,.\,0_{13,33} + \beta\,.\,0_{13,24} \quad . \quad . \quad . \quad . \quad (117)$$

und jedes J_{11} die Gestalt

$$J_{11} = aS + bR' + cQ'' + d.0_{13,25} = \alpha.0_{23,33} + \beta.0_{13,34} + \gamma.0_{13,25} + \delta.0_{23,24} \quad (118)$$

mit konstanten Koeffizienten a, b, c, d bzw. α, β, γ und δ.

Darüber hinaus ist leicht zu beweisen, dass die angeschriebenen Ausdrücke für J_{10} und J_{11} nur dann verschwinden (für alle Parameterwerte t), wenn alle Koeffizienten a bis d Null sind. Das bedeutet die lineare Unabhängigkeit von R und Q' und die von S, R', Q'' und $0_{13,25}$. Zum Beweise kann man von $0_{ik} = (yz)_{ik}$ ausgehen, was wir aber hier nicht weiter ausführen wollen.

Für die in (118) rechter Hand stehenden Ausdrücke ergeben sich dagegen bei Benutzung der Identität (91) quadratische Beziehungen. Man erhält:

$$0_{13,34}\,.\,0_{23,33} = -0_{13,23}\,.\,0_{33,34} - 0_{13,33}\,.\,0_{34,23}$$

$$\tfrac{1}{2}(R' + S)\,.\,S = -Q\,.\,0_{33,34} + R\,.\,0_{23,34} \quad . \quad . \quad . \quad . \quad (119)$$

$$0_{13,34}\,.\,0_{23,24} = -0_{13,23}\,.\,0_{24,34} - 0_{13,24}\,.\,0_{34,23}$$

$$\tfrac{1}{2}(R' + S)\,.\,0_{23,24} = -Q\,.\,0_{24,34} + (Q' - \tfrac{5}{6} R)\,.\,0_{23,34} \quad . \quad . \quad (120)$$

$$0_{13,25}\,.\,0_{23,33} = -0_{13,23}\,.\,0_{33,25} - 0_{13,33}\,.\,0_{25,23}$$

$$S\,.\,0_{13,25} = -Q\,.\,0_{33,25} + R\,.\,0_{23,25} \quad . \quad . \quad . \quad . \quad . \quad (121)$$

$$0_{13,25}\,.\,0_{23,24} = -0_{13,23}\,.\,0_{24,25} - 0_{13,24}\,.\,0_{25,23}$$

$$0_{13,25}\,.\,0_{23,24} = -Q\,.\,0_{24,25} + (Q' - \tfrac{5}{6} R)\,.\,0_{23,25} \quad . \quad . \quad (121a)$$

Nach (116) kann man dieser letzten Gleichung die Gestalt geben

$$\tfrac{2}{3}(0_{13,25})^2 - 0_{13,25}\,.\,(\tfrac{2}{3}Q'' - \tfrac{7}{9}R') = -Q\,.\,0_{24,25} + (Q' - \tfrac{5}{6}R)\,.\,0_{23,25} \quad (121b)$$

Aus (119) folgt: Sind Q und $R \equiv 0$, so ist auch $S \equiv 0$. Ferner aus (121b): $Q \equiv 0$, $R \equiv 0$ zieht $0_{13,25} \equiv 0$ und damit nach (116) auch $0_{23,24} \equiv 0$ nach sich. *Also verschwinden dann alle Ausdrücke J_{10} und J_{11}.*

§ 14.

In Gleichung (43) wird die Invariante U durch Formen J_{10} und J_{11} ausgedrückt:

$$\left.\begin{aligned} U = (\tfrac{1}{2}R + 3Q')\,.\,(0_{22,33} + 3\,.\,0_{22,24} + 3\,.\,0_{22,15}) + \\ + 4Q\,.\,(2_{03,33} + 3\,.\,2_{03,24} + 3\,.\,2_{03,15} + 2_{03,06}) - \\ - 12Q\,.\,(0_{23,33} + 3\,.\,0_{23,24} + 3\,.\,0_{23,15}) \end{aligned}\right\} \quad . \quad (122)$$

Nach § 12 erhalten wir hier:

$$0_{22,33} = -\frac{4}{3}R,\; 0_{22,24} = -\frac{4}{3}\,.\,0_{13,24} = \frac{10}{9}R - \frac{4}{3}Q',\; 0_{22,15} = -\frac{5}{9}R + \frac{10}{3}Q' \quad (123)$$

$$2_{03,33} = -S,\; 2_{03,24} = \frac{1}{3}R' + S,\; 2_{03,15} = 0_{13,25},\; 2_{03,06} = -5R' - 5S - 6.0_{13,25} \quad (124)$$

$$\left.\begin{aligned} 0_{23,33} = S \quad , \quad 0_{23,24} = \frac{7}{9}R' - \frac{2}{3}Q'' + \frac{2}{3}\,.\,0_{13,25} \\ 0_{23,15} = -\frac{5}{3}S - \frac{35}{18}R' + \frac{5}{3}Q'' - \frac{5}{3}\,.\,0_{13,25}. \end{aligned}\right\} \quad . \quad . \quad (125)$$

Setzt man dies alles in (122) ein, so kommt:

$$U = \frac{1}{6}R^2 + 4Q'R + 18Q'^2 - 36QQ'' + 26QR' + 36QS + 24Q\,.\,0_{13,25}. \quad (126)$$

Diese Invariante hat die Gewichte $\varrho = 20$, $\sigma = 10$. Sie ist (nach Q) aber nicht die einfachste Differentialinvariante, wie wir jetzt beweisen wollen.

Zu diesem Zwecke schreiben wir mit Hilfe der Gleichungen (12) und (14) die zu den Transformationen

$$t = \varphi(\tilde{t}) \;,\; \widehat{a_{ik}} = \lambda(t)\,.\,a_{ik}$$

gehörigen Transformationsgleichungen für die in (126) rechts stehenden Ausdrücke

$$Q, Q', Q'', R, R', S \text{ und } S_0 = 0_{13,25}$$

auf. Was die zuletzt genannte Grösse S_0 betrifft, hat man vorerst die Gleichungen (12) und (14) zu ergänzen mit den beiden folgenden

$$\left.\begin{aligned}
\widetilde{M}'_{25} = {} & M'_{25}\,.\,\varphi'^7 - M'_{33}\,.\,\tfrac{5}{3}\,\varphi'^5\,\varphi'' + M'_{24}\,.\,\tfrac{15}{2}\,\varphi'^5\,\varphi'' - M_{06}\,.\,\tfrac{1}{6}\,\varphi'^5\,\varphi'' + \\
& + M'_{23}\,.\,(-5\,\varphi'^3\,\varphi''^2 + 10\,\varphi'^4\,\varphi''') - \\
& - M'_{05}\,.\,2\,\varphi'^3\,\varphi''^2 + M'_{13}\,.\,(15\,\varphi'\,\varphi''^3 - \tfrac{10}{3}\,\varphi'^2\,\varphi''\,\varphi''') - \\
& - M'_{04}\,.\,(\tfrac{10}{3}\,\varphi'^2\,\varphi''\,\varphi''' + \tfrac{5}{3}\,\varphi'^3\,\varphi^{IV}) - \\
& - M'_{03}\,.\,\tfrac{1}{3}\,(\varphi'^2\,\varphi^{V} + 10\,\varphi''^2\,\varphi''' + 5\,\varphi'\,\varphi''\,\varphi^{IV}) - M'_{02}\,.\,\varphi''\,\varphi^{V}.
\end{aligned}\right\}\quad (127)$$

$$\left.\begin{aligned}
\widehat{M}'_{25} = {} & M'_{25}\,.\,\lambda^2 - M'_{33}\,.\,\tfrac{10}{3}\,\lambda\lambda' - M'_{06}\,.\,\tfrac{1}{3}\,\lambda\lambda' + M'_{23}\,.\,(10\,\lambda\lambda'' - 20\,\lambda'^2) + \\
& + M'_{05}\,(\lambda\lambda'' - 2\,\lambda'^2) + \\
& + M'_{13}\,.\,(20\,\lambda'\,\lambda'' - \tfrac{40}{3}\,\lambda\lambda''') + M'_{04}\,.\,(5\,\lambda'\,\lambda'' - \tfrac{10}{3}\,\lambda\lambda''') + \\
& + M'_{03}\,.\,(10\,\lambda''^2 - \tfrac{5}{3}\,\lambda\lambda^{IV} - \tfrac{20}{3}\,\lambda'\,\lambda''') + \\
& + M'_{02}\,.\,(10\,\lambda''\,\lambda''' - 10\,\lambda'\,\lambda^{IV} + \lambda\lambda^{V}).
\end{aligned}\right\}\quad (128)$$

Wir haben dann bei $t \to \widetilde{t}$:

$$\left.\begin{aligned}
\widetilde{Q}'' &= Q''\,.\,\varphi'^{11} + Q'\,.\,19\,\varphi'^9\,\varphi'' + Q\,.\,(72\,\varphi'^7\,\varphi''^2 + 9\,\varphi'^8\,\varphi''') \\
\widetilde{R}' &= R'\,.\,\varphi'^{11} + R\,.\,10\,\varphi'^9\,\varphi'' + Q'\,.\,6\,\varphi'^9\,\varphi'' + Q\,(48\,\varphi'^7\,\varphi''^2 + 6\,\varphi'^8\,\varphi''') \\
\widetilde{S} &= S\,.\,\varphi'^{11} - R\,.\,3\,\varphi'^9\,\varphi'' - Q\,.\,(6\,\varphi'^7\,\varphi''^2 + 2\,\varphi'^8\,\varphi''') \\
\widetilde{S}_0 &= S_0\,.\,\varphi'^{11} - R\,.\,\tfrac{95}{12}\,\varphi'^9\,\varphi'' + Q'\,.\,\tfrac{15}{2}\,\varphi'^9\,.\,\varphi'' + \\
& \qquad + Q\,.\,(-5\,\varphi'^7\,\varphi''^2 + 10\,\varphi'^8\,\varphi''') \\
\widetilde{R}^2 &= R^2\,.\,\varphi'^{20} + RQ\,.\,12\,\varphi'^{18}\,\varphi'' + Q^2\,.\,36\,\varphi'^{16}\,\varphi''^2 \\
\widetilde{Q}'\,\widetilde{R} &= Q'\,R\,.\,\varphi'^{20} + RQ\,.\,9\,\varphi'^{18}\,\varphi'' + QQ'\,.\,6\,\varphi'^{18}\,\varphi'' + Q^2\,.\,54\,\varphi'^{16}\,\varphi''^2 \\
\widetilde{Q}'^2 &= Q'^2\,.\,\varphi'^{20} + QQ'\,.\,18\,\varphi'^{18}\,\varphi'' + Q^2\,.\,81\,\varphi'^{16}\,\varphi''^2
\end{aligned}\right\}\quad (129)$$

und ebenso bei $a_{ik} \to \widehat{a}_{ik}$:

$$\left.\begin{aligned}
\widehat{Q}'' &= Q''\,.\,\lambda^5 + Q'\,.\,10\,\lambda^4\,\lambda' + Q\,.\,(20\,\lambda^3\,\lambda'^2 + 5\,\lambda^4\,\lambda'') \\
\widehat{R}' &= R'\,.\,\lambda^5 + R\,.\,5\,\lambda^4\,\lambda' + Q'\,.\,6\,\lambda^4\,\lambda' + Q\,(24\,\lambda^3\,\lambda'^2 + 6\,\lambda^4\,\lambda'') \\
\widehat{S} &= S\,.\,\lambda^5 - R\,.\,2\,\lambda^4\,\lambda' - Q\,.\,6\,\lambda^4\,\lambda'' \\
\widehat{S}_0 &= S_0\,.\,\lambda^5 - R\,.\,\tfrac{10}{3}\,\lambda^4\,\lambda' + Q\,.\,(10\,\lambda^4\,\lambda'' - 20\,\lambda'^2) \\
\widehat{R}^2 &= R^2\,.\,\lambda^{10} + RQ\,.\,12\,\lambda^9\,\lambda' + Q^2\,.\,36\,\lambda^8\,\lambda'^2 \\
\widehat{Q}'\,\widehat{R} &= Q'\,R\,.\,\lambda^{10} + RQ\,.\,5\,\lambda^9\,\lambda' + QQ'\,.\,6\,\lambda^9\,\lambda' + Q^2\,.\,30\,\lambda^8\,\lambda'^2 \\
\widehat{Q}'^2 &= Q'^2\,.\,\lambda^{10} + QQ'\,.\,10\,\lambda^9\,\lambda' + Q^2\,.\,25\,\lambda^8\,\lambda^{12}
\end{aligned}\right\}\quad (130)$$

Jede Differentialinvariante vom φ'-Gewichte elf entsteht aus den ersten vier der Gleichungen (129) durch Elimination von φ''' und φ'' und φ''^2, und zwar durch lineare Kombination mit konstanten Koeffizienten x, y, z und s, d.h.

$$\widetilde{J} = x\widetilde{Q}'' + y\widetilde{R}' + z\widetilde{S} + s\widetilde{S}_0$$

muss ohne φ'', φ''^2 und φ''' sein. Ebenso darf

$$\widehat{J} = x\widehat{Q}'' + y\widehat{R}' + z\widehat{S} + s\widehat{S}_0$$

kein λ' und λ'' enthalten. Diese Forderungen führen auf ein System linearer Gleichungen für x, y, z und s, das nur die Lösung 0, 0, 0, 0 hat.

Der nächst-einfachere Fall bringt dieselben Ueberlegungen für etwaige Invarianten vom φ'-Gewichte 20. Hier sind dann sieben homogene Konstanten x, y, z, s, a, b und c zu ermitteln, sodass

$$\widetilde{J} = x\widetilde{Q}\widetilde{Q}'' + y\widetilde{Q}\widetilde{R}' + z\widetilde{Q}\widetilde{S} + s\widetilde{Q}\widetilde{S}_0 + a\widetilde{R}^2 + b\widetilde{Q}'\widetilde{R} + c\widetilde{Q}'^2 \quad (131)$$

kein φ'', φ''^2 und φ''' enthält und analog $\widehat{J}$ ohne λ' und λ'' ist. Die Koeffizienten dieser Grössen finden wir in der aus (129) und (130) entnommenen Tabelle

	$\varphi'^{18}\varphi''$	$\varphi'^{16}\varphi''^2$	$\varphi'^{17}\varphi'''$		$\lambda^9\lambda'$	$\lambda^8\lambda'^2$	$\lambda^9\lambda''$
QQ''	$19\,QQ'$	$72\,Q^2$	$9\,Q^2$	x	$10\,QQ'$	$20\,Q^2$	$5\,Q^2$
QR'	$10\,QR+6\,QQ'$	$48\,Q^2$	$6\,Q^2$	y	$5\,RQ+6\,QQ'$	$24\,Q^2$	$6\,Q^2$
QS	$-3\,RQ$	$-6\,Q^2$	$-2\,Q^2$	z	$-2\,RQ$	0	$-6\,Q^2$
QS_0	$-\frac{95}{12}RQ+\frac{15}{2}QQ'$	$-5Q^2$	$10\,Q^2$	s	$-\frac{10}{3}RQ$	$-20\,Q^2$	$10\,Q^2$
R^2	$12\,RQ$	$36\,Q^2$	0	a	$12\,RQ$	$36\,Q^2$	0
RQ'	$9\,RQ+6\,QQ'$	$54\,Q^2$	0	b	$5\,RQ+6\,QQ'$	$30\,Q^2$	0
Q'^2	$18\,QQ'$	$81\,Q^2$	0	c	$10\,QQ'$	$25\,Q^2$	0

Hieraus ergeben sich die Gleichungen

$$\left.\begin{array}{ll}
1a)\ 19x+6y+\frac{15}{2}s+6b+18c=0 & 2)\ 72x+48y-6z-5s+36a+54b+81c=0 \\[2ex]
1b)\ 10y-3z-\frac{95}{12}s+12a+9b=0 & 3)\ 9x+6y-2z+10s=0 \\[2ex]
4a)\ 5y-2z-\frac{10}{3}s+12a+5b=0 & 5)\ 20x+24y-20s+36a+30b+25c=0 \\[2ex]
4b)\ 10x+6y+6b+10c=0 & 6)\ 5x+6y-6z+10s=0
\end{array}\right\} \quad (132)$$

Berechnen wir hier aus 1*a*), 1*b*) und 4*b*) die Grössen *a*, *b* und *c* und setzen die gefundenen Ausdrücke in 4*a*), 2) und 5) ein, so ergeben sich die Gleichungen 3) und 6). Alle acht Gleichungen sind demnach äquivalent mit

$$11x + 6y + 30s = 0 \quad \text{und} \quad z = -x.$$

Damit nimmt die allgemeinste, aus (131) entspringende Invariante J die Gestalt an

$$\left.\begin{aligned} J_{20} = &\left(-\frac{73}{288}R^2 + \frac{49}{24}Q'R - \frac{9}{8}Q'^2 + QQ'' - \frac{11}{6}QR' - QS\right).x + \\ &+\left(-\frac{215}{576}R^2 + \frac{155}{48}Q'R - \frac{15}{16}Q'^2 - \frac{5}{3}QR' + QS_0\right).s \end{aligned}\right\} \quad (133)$$

wobei x und s beliebige Zahlenkoeffizienten bedeuten.

Wir setzen

$$J_{20} = x.A + s.B \quad \ldots\ldots\ldots \quad (134)$$

wo A und B aus (133) zu entnehmen sind.

Man beweist leicht, dass weder A noch B identisch Null und dass beide linear-unabhängig von einander sind.

Den Werten $x = -36$ und $s = 24$ entspricht die Invariante U von (126):

$$U = -36A + 24B. \quad \ldots\ldots\ldots \quad (135)$$

Mathematics. — *Zur projektiven Differentialgeometrie der Regelflächen im R_4.* (Fünfte Mitteilung). Von R. WEITZENBÖCK.

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Mit Hilfe des Heftpunktes H und des invarianten Punktes F lässt sich für die Erzeugende 0_{ik} der Fläche eine invariante Darstellung geben. Dies ist auch möglich, wenn man den Punkt M und dessen Ableitungen benutzt. Hier erhält man dann auch invariante Darstellungen, wenn man die gewöhnlichen Ableitungen durch kovariante Ableitungen ersetzt. Dies führt im Falle einer allgemeinen Fläche zur Beantwortung der Frage nach einem *wesentlichen System von Differentialinvarianten.* Man erhält deren sieben, von denen bei invarianter Normierung noch zwei in Wegfall kommen.

§ 15.

Wenn wir den durch $(xM'_{02})=0$ gegebenen Tangentialraum kurz durch die Reihe M' andeuten, also

$$(xM'_{02})=(x\,0^2\,2^2)=(xM') \quad \ldots \ldots \quad (136)$$

setzen, und wenn die Ableitungen von M' durch

$$M'_1=\frac{dM'}{dt}\,,\; M'_2=\frac{d^2M'}{dt^2},\ldots$$

festgehalten werden, so ist nach (1):

$$(xM'_1)=(xM'_{12})+(xM'_{03})=\tfrac{2}{3}(xM'_{03})$$

$$(xM'_2)=\tfrac{2}{3}\left[(xM'_{13})+(xM'_{04})\right],\ \text{u. s. w.}$$

Die Heftebene m_{ijk} erscheint dann nach (32) dargestellt durch

$$m'_{ik}=(M'_{02}\,M'_{03})_{ik}=\tfrac{3}{2}(M'\,M'_1)_{ik}. \quad \ldots \ldots \quad (137)$$

Die Gerade m_{ik} von (37) ist die Schnittgerade dreier aufeinanderfolgender Tangentialräume und erhält jetzt die Gestalt

$$m_{rs}=-\tfrac{3}{4}(M'\,M'_1\,M'_2)_{ijk}. \quad \ldots \ldots \quad (138)$$

Den Schnittpunkt von vier konsekutiven Tangentialräumen haben wir M genannt (vgl. in § 5 die Gleichungen (39) und (42)). Mit Hilfe der Reihe M' haben wir

$$(Mu')=\tfrac{9}{8}(u'\,M'\,M'_1\,M'_2\,M'_3), \quad \ldots \ldots \quad (139)$$

wobei (Mu') nach (42) und (82) gegeben ist durch

$$(Mu') = (\tfrac{1}{2} R + 3Q') . (Hu') + 4Q . [(2u') 2_{03} - 3 . (0u') 0_{23}]. \quad (140)$$

Setzen wir hier

$$u' = M_4' = \tfrac{8}{3} (M_{33}' + 3M_{24}' + 3M_{15}' + M_{06}'),$$

so entsteht nach (43) die Invariante U. Wir haben

$$\triangle_{M'} = (M' M_1' M_2' M_3' M_4') = \tfrac{8}{9} (MM_4') = \tfrac{16}{27} U. \quad . \quad . \quad (141)$$

Geht man umgekehrt vom Punkte M aus und setzt

$$M_1 = \frac{dM}{dt}, \quad M_2 = \frac{d^2 M}{dt^2}, \ldots$$

so findet man:

$$(MM_1)_{ik} = \tfrac{3}{4} U . (M' M_1' M_2')_{rst} = -U . m_{ik} \quad . \quad . \quad . \quad (142)$$

$$(MM_1 M_2)_{ijk} = \tfrac{1}{2} U^2 . (M' M_1')_{rs} \quad . \quad . \quad . \quad . \quad . \quad (143)$$

$$(MM_1 M_2 M_3)_i = -\tfrac{1}{3} U^3 . M_i' \quad . \quad . \quad . \quad . \quad . \quad (144)$$

$$\triangle_M = (MM_1 M_2 M_3 M_4) = \tfrac{2}{9} U^4 = \frac{3^{10}}{2^{15}} . \triangle_{M'}^4 \quad . \quad . \quad . \quad (145)$$

Diese Formeln zeigen: Bei $U \neq 0$ ist es für die Untersuchung der Kurve C_M = Ort der Punkte M gleichgiltig ob man C_M als Punktort oder als Ort der Schmieg-R_3 M' auffasst.

§ 16.

Nach (142) ist die Gerade, die die Punkte M und $M_1 = \dfrac{dM}{dt}$ verbindet, mit m_{ik} identisch, geht also durch den Heftpunkt H. Es muss also zwischen den drei Punkten M, M_1 und H eine lineare Relation bestehen, die wir jetzt ableiten wollen.

Im Ansatze

$$\tau . (Mu') + (M_1 u') \equiv \varepsilon . (Hu') \qquad \text{(identisch in } u') \quad . \quad . \quad (146)$$

ermitteln wir zuerst τ durch die Substitution $u' = M_{13}'$. Führt man diese Substitution in (140) und der daraus durch Differentiation entstehenden Gleichung für $(M_1 u')$ aus, so ergibt sich nach leichter Rechnung

$$(MM_{13}') = -16 Q^2, \quad (HM_{13}') = 0$$

$$(M_1 M_{13}') = -24 QQ' - \tfrac{4}{3} QR. \quad . \quad . \quad . \quad . \quad . \quad (147)$$

Also wird bei $Q \neq 0$:

$$4Q \,.\, \tau = -6Q' - \tfrac{1}{3} R,$$

d.h. (146) bekommt die Gestalt

$$-(6Q' + \tfrac{1}{3} R) \,.\, M + 4Q \,.\, M_1 \equiv \varepsilon \,.\, 4Q \,.\, H.$$

Zur Bestimmung von ε setzen wir jetzt $u' = M'_{23}$. Es wird

$$\left.\begin{aligned} (MM'_{23}) &= \tfrac{4}{3} Q(-\tfrac{1}{2} R + 3Q') \;,\; (HM'_{23}) = \tfrac{4}{3} Q \\ (M_1 M'_{23}) &= (\tfrac{1}{2} R' + 3Q'') \,.\, \tfrac{4}{3} Q - \tfrac{4}{3} Q' R - \\ &- 12\,QS - 12\,Q\,0_{23,24} - \tfrac{2}{9} R \,.\, (\tfrac{1}{2} R + 3Q'). \end{aligned}\right\} \quad . \quad . \quad (148)$$

Dies gibt nach (125) und (126)

$$4Q\varepsilon = -U$$

und damit erhalten wir schliesslich für (146):

$$(6Q' + \tfrac{1}{3} R) \,.\, M - 4Q \,.\, M_1 = U \,.\, H. \quad . \quad . \quad . \quad . \quad (149)$$

Nach Gleichung (143) liegt, wieder für $U \neq 0$, der Punkt M_2 in der Heftebene m'_{rs}. Es müssen sich also die Punkte H_1 (Gleichung (54)), G (Gleichung (95)) und F (Gleichung (96)) als Linearkombinationen von M, M_1 und M_2 darstellen lassen. Wir sahen am Ende des § 11, dass die Gerade MG die Tangente h_{ik} im Punkte $M + 3G$ schneidet. Daher müssen sich α und β so bestimmen lassen, dass

$$(Mu') + 3(Gu') \equiv \alpha \,.\, (Hu') + \beta \,.\, (H_1 u')$$

wird. Setzt man hier wieder für u' die Reihen M'_{13} und M'_{23} ein, wobei man für G den Ausdruck (95) benutzt, so entsteht

$$2M + 6G = (6Q' + 7R) \,.\, H - 24\,Q \,.\, H_1. \quad . \quad . \quad . \quad (150)$$

Differenzieren wir jetzt (149), so kommt:

$$(6Q'' + \tfrac{1}{3} R') \,.\, M + (2Q' + \tfrac{1}{3} R) \,.\, M_1 - 4QM_2 = U' \,.\, H + U \,.\, H_1. \quad (151)$$

Hier kann man mit Hilfe von (149) H eliminieren und erhält

$$\left.\begin{aligned} U^2 \,.\, H_1 &= (6UQ'' + \tfrac{1}{3} UR' - 6U'Q' - \tfrac{1}{3} U'R) \,.\, M + \\ &+ (2Q'U + 4QU' + \tfrac{1}{3} UR) \,.\, M_1 - 4QU \,.\, M_2. \end{aligned}\right\} . \quad (152)$$

Aus (150) findet man hiermit

$$\left.\begin{aligned} U^2 \,.\, G &= [-\tfrac{1}{3} U^2 + U(6Q' + 7R)(Q' + \tfrac{1}{18} R) - \\ &- 4Q(6UQ'' + \tfrac{1}{3} UR' - 6U'Q' - \tfrac{1}{3} U'R)] \,.\, M - \\ &- (12\,QQ'U + 16\,Q^2 U' + 6\,QRU) \,.\, M_1 + 16\,Q^2 U \,.\, M_2 \end{aligned}\right\} \quad (153)$$

Wegen $F = M + G$ folgt hieraus

$$\left.\begin{aligned} U^2 . F = [\tfrac{2}{3} U^2 + U(6Q' + 7R)(Q' + \tfrac{1}{18} R) - \\ -4Q(6UQ'' + \tfrac{1}{3} UR' - 6U'Q' - \tfrac{1}{3} U'R)] . M - \\ -(12QQ'U + 16Q^2U' + 6QRU) . M_1 + 16Q^2U . M_2 . \end{aligned}\right\} \quad (154)$$

Der Punkt F liegt auf der Erzeugenden 0_{ik}. Aus

$$F = (\tfrac{3}{2} R + 3Q') . H - 16Q . (0u') 0_{23} \qquad H = (0u') 0_{22}$$

finden wir dann

$$(HF)_{ik} = -16Q (0\dot{0})_{ik} 0_{22} \dot{0}_{23} = -16Q . 0_{ik} . -\tfrac{4}{3} Q,$$

$$(HF)_{ik} = \frac{64}{3} Q^2 . 0_{ik} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

Berechnen wir dann $(HF)_{ik}$ nach (149) und (154), so erscheint 0_{ik} als Gerade der Heftebene (MM_1M_2) wie folgt dargestellt:

$$\left.\begin{aligned} \frac{64}{3} U^3 Q . 0_{ik} = -(M_1M_2)_{ik} . 64Q^2U - (M_2M)_{ik} . 16QU(6Q' + \tfrac{1}{3} R) + \\ + (MM_1)_{ik} . 2\{-(6Q'U + 8QU' + 3RU)(6Q' + \tfrac{1}{3} R) + \\ + 2 . [\tfrac{2}{3} U^2 + U(6Q' + 7R)(Q' + \tfrac{1}{18} R)] - 8Q[6Q''U + \\ + \tfrac{1}{3} R'U - 6Q'U' - \tfrac{1}{3} RU']\} \end{aligned}\right\} (156)$$

§ 17.

Die Ausdrücke M_1 und M_2 sind keine projektiven Differentialkontravarianten; demzufolge sind die Koeffizienten von $(M_2M)_{ik}$ und von $(MM_1)_{ik}$ in der letzten Gleichung keine Invarianten. Um eine invariante Darstellung von 0_{ik} in der Heftebene zu erhalten, müssen wir statt M_1 und M_2 invariante Punkte $M_{(1)}$ und $M_{(2)}$ wählen. Dies kommt darauf hinaus: die gewöhnlichen Ableitungen nach dem Parameter t sind zu ersetzen durch kovariante Ableitungen.

Dies kann man auf verschiedene Arten tun. Auf der Hand liegt, die Formel (18) zu benutzen. $M = (Mu')$ ist vom φ'-Gewichte 14 und vom λ-Gewichte 8. Sind dann J_1, J_2 zwei Differentialinvarianten mit den Gewichten (ϱ_1, σ_1) bzw. (ϱ_2, σ_2), so wird

$$\{M, J_1, J_2\} = \begin{vmatrix} M_1 & J_1' & J_2' \\ 14M & \varrho_1 J_1 & \varrho_2 J_2 \\ 8M & \sigma_1 J_1 & \sigma_2 J_2 \end{vmatrix} = M . [(14\sigma_1 - 8\varrho_1) J_1 J_2' - (14\sigma_2 - 8\varrho_2) J_1' J_2] + \\ + M_1 . (\varrho_1\sigma_2 - \varrho_2\sigma_1) J_1 J_2$$

ein invarianter Punkt der Geraden MM_1.

Wählt man hier z.B. als einfachste Invarianten J:

$$J_1 = Q \text{ mit } \varrho_1 = 9 \quad , \quad \sigma_1 = 5 \text{ und } J_2 = U \text{ mit } \varrho_2 = 20 \quad , \quad \sigma_2 = 10,$$

so ergibt sich aus einer Komitante $K(\varrho, \sigma)$ die „kovariante Ableitung" K_I:

$$K_I = \{K, Q, U\} = -10\, QUK_1 + [(5\varrho - 9\sigma)\, QU' - (10\varrho - 20\sigma)\, Q'U] \,.\, K. \quad (157)$$

Dies gibt für $K = M$ den Punkt

$$M_I = (20\, Q'U - 2\, QU') \,.\, M - 10\, QU \,.\, M_1. \quad . \quad . \quad . \quad (158)$$

Es geht aber noch einfacher. Wir haben nämlich die Transformationsgleichungen

$$\widetilde{M} = M \,.\, \varphi'^{14} \qquad \widehat{M} = M \,.\, \lambda^8$$

$$\widetilde{M}_1 = M_1 \,.\, \varphi'^{15} + M \,.\, 14 \varphi'^{13} \varphi'' \qquad \widehat{M}_1 = M_1 \,.\, \lambda^8 + M \,.\, 8 \lambda^7 \lambda'$$

Nehmen wir hierzu:

$$\widetilde{Q}' = Q' \,.\, \varphi'^{10} + Q \,.\, 9 \varphi'^8 \varphi'' \qquad \widehat{Q}' = Q' \,.\, \lambda^5 + Q \,.\, 5 \lambda^4 \lambda'$$

$$\widetilde{R} = R \,.\, \varphi'^{10} + Q \,.\, 6 \varphi'^8 \varphi'' \qquad \widehat{R} = R \,.\, \lambda^5 + Q \,.\, 6 \lambda^4 \lambda'$$

dann lassen sich in

$$M_{(1)} = QM_1 + (aR + bQ') \,.\, M$$

die Konstanten a und b so wählen, dass $\widetilde{M}_{(1)}$ frei von φ'' und $\widehat{M}_1$ frei von λ' wird. Man erhält

$$M_{(1)} = 24\, Q \,.\, M_1 - 2\,(R + 18\, Q') \,.\, M \quad . \quad . \quad . \quad . \quad . \quad (159)$$

und auch dies kann man eine „kovariante Ableitung" nennen.

Allgemein erhält man auf diese Weise bei einer Komitante K mit den Gewichten ϱ und σ die kovariante Ableitung

$$K_{(1)} = 24\, Q \,.\, K_1 + [(5\varrho - 9\sigma)\, R - 6\,(\varrho - \sigma)\, Q'] \,.\, K; \quad . \quad . \quad (160)$$

$K_{(1)}$ hat die Gewichte $\varrho + 10$ und $\sigma + 5$.

Vergleichen wir (159) mit (149), so finden wir

$$M_{(1)} = -(36\, Q' + 2\, R) \,.\, M + 24\, Q \,.\, M_1 = -6\, U \,.\, H, \quad . \quad (161)$$

d. h. $M_{(1)}$ führt auf den Punkt H.

Die kovariante Ableitung $Q_{(1)}$ von Q selbst verschwindet identisch, wie man aus (160) für $Q = K$ sofort feststellt. Bei der Invariante U dagegen erhält man

$$U_{(1)} = 24\, QU' + (10\, R - 60\, Q')\, U = 24\, V. \quad . \quad . \quad . \quad (162)$$

Ist $K = J \,.\, L$, so gilt nach (160) die Differentiationsregel

$$(J \,.\, L)_{(1)} = J_{(1)} L + J L_{(1)}.$$

Wir übertragen nun die Gleichung (156) auf die Punkte M, $M_{(1)}$ und $M_{(2)}$, wobei $M_{(2)} = (M_{(1)})_{(1)}$ gesetzt ist („zweite" kovariante Ableitung). Wir haben nach (160) und (150)

$$H_{(1)} = 24\, Q \,.\, H_1 - (7\, R + 6\, Q') \,.\, H = -2\, M - 6\, G \quad . \quad . \quad (163)$$

und aus (161):

$$M_{(2)} = -6\, U_{(1)} \,.\, H - 6\, U \,.\, H_{(1)}.$$

Also wird, wenn wir $H_{(1)}$ eliminieren, analog mit (153):

$$36\, U \,.\, G = -12\, U \,.\, M + 6\, U_{(1)} \,.\, H + M_{(2)},$$

somit nach (161):

$$36\, U^2 \,.\, G = -12\, U^2 \,.\, M + U \,.\, M_{(2)} - U_{(1)} \,.\, M_{(1)}. \quad . \quad . \quad (164)$$

Hieraus, analog zu (154):

$$36\, U^2 \,.\, F = 24\, U^2 \,.\, M + U \,.\, M_{(2)} - U_{(1)} \,.\, M_{(1)}, \quad . \quad . \quad . \quad (165)$$

und hieraus schliesslich nach (155) bei $U \neq 0$:

$$72 \,.\, 64 \,.\, Q^2\, U^2 \,.\, 0_{ik} = 24\, U \,.\, (M M_{(1)})_{ik} - (M_{(1)}\, M_{(2)})_{ik} \,. \quad . \quad (166)$$

Wir benötigen auch die entsprechende Gleichung für die durch römische Ziffern festgehaltenen kovarianten Ableitungen M_I, $M_{II}, \ldots$ (Vgl. Gleichung (158)). Zunächst bemerken wir, dass nach (157)

$$Q_I = 0 \quad \text{und} \quad U_I = 0$$

gilt. Dann wird, wenn wir M_1 aus (158) und (161) eliminieren:

$$M_I = \tfrac{5}{2}\, U^2 \,.\, H - (2\, Q U' - 5\, Q' U + \tfrac{5}{6}\, R U) \,.\, M, \quad . \quad . \quad (167)$$

oder auch, nach (162):

$$M_I = \tfrac{5}{2}\, U^2 \,.\, H - 2\, V \,.\, M \quad \text{mit} \quad V = Q U' + \tfrac{5}{12}\, R U - \tfrac{5}{2}\, Q' U. \quad (167\text{a})$$

Nach (157) haben wir für $K = H$:

$$H_I = -10\, Q U \,.\, H_1 + (20\, Q' U - 7\, Q U') \,.\, H. \quad . \quad . \quad . \quad (168)$$

Eliminieren wir mit Hilfe von (163) H_1, so kommt

$$H_I = \tfrac{5}{6}\, U \,.\, M + \tfrac{5}{2}\, U \,.\, G - 7\, V \,.\, H. \quad . \quad . \quad . \quad . \quad (169)$$

Weiters ergibt sich durch kovariantes Differentieren aus (167*a*):

$$M_{II} = \tfrac{5}{2}\, U^2 \,.\, H_I - 2\, V_I \,.\, M - 2\, V \,.\, M_I .$$

Setzen wir hier (169) ein, so entsteht

$$\frac{25}{4}\, U^3 \,.\, G = M_{II} + 2\, V \,.\, M_I + \left(2\, V_I - \frac{25}{12}\, U^3\right) . M + \frac{35}{2}\, U^2\, V \,.\, H. \quad (170)$$

Hierin ist nach (157) für $K = V$:

$$V_I = 5\, Q\, (3\, U'\, V - 2\, U V') = 5\, Q \,.\, W, \quad . \; . \; . \; . \quad (171)$$

d. h. V_I zerfällt in Q und die Invariante

$$W = 3\, U' V - 2\, U V' ; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (172)$$

W hat die Gewichte $\varrho = 51$ und $\sigma = 25$. Statt (170) haben wir dann auch

$$\frac{25}{4}\, U^3 . G = M_{II} + 2\, V \,.\, M_I + \left(10\, Q W - \frac{25}{12}\, U^3\right) . M + \frac{35}{2}\, U^2\, V . H. \quad (173)$$

Hieraus weiter für $F = M + G$:

$$\frac{25}{4}\, U^3 . F = M_{II} + 2\, V . M_I + \left(10\, Q W + \frac{25}{6}\, U^3\right) . M + \frac{35}{2}\, U^2\, V . H. \quad (174)$$

Mit Hilfe von (155) finden wir also schliesslich nach (167a):

$$\left.\begin{aligned} \frac{10^3}{3}\, Q^2\, U^5 \,.\, 0_{ik} &= (M_I\, M_{II})_{ik} - 2\, V \,.\, (M_{II}\, M)_{ik} + \\ &+ \left[4\, V^2 - 10\, Q W - \frac{25}{6}\, U^3\right] . (M M_I)_{ik}. \end{aligned}\right\} \quad . \; . \; . \quad (175)$$

Hiermit ist also die Erzeugende 0_{ik} auch durch die drei Seiten des Dreieckes $M M_I\, M_{II}$ ausgedrückt.

§ 18.

Die Gleichung (175) gestattet es, auf die Frage nach den *wesentlichen Differentialinvarianten* eine Antwort zu geben, wobei wir den sogenannten allgemeinen Fall voraussetzen, in dem keine der Invarianten Q und U identisch Null ist. Ein wesentliches System von Differentialinvarianten $J_1, J_2, \ldots, J_m$ liegt bekanntlich vor, wenn jede weitere Differentialinvariante von F durch diese Invarianten J_s und durch deren Ableitungen $\frac{d J_s}{d t}, \frac{d^2 J_s}{d t^2}, \ldots$ ausgedrückt werden kann.

Auf Grund der quinären Identität

$$(01234)\,.\,5=(12345).0-(02345).1+(01345).2-(01245).3+(01235).4 \quad (176a)$$

oder kürzer

$$C_5\,.\,5=C_0\,.\,0-C_1\,.\,1+C_2\,.\,2-C_3\,.\,3+C_4\,.\,4 \quad . \quad . \quad (176b)$$

kann bei $C_5 \neq 0$ der Punkt 5 als Linearkombination der Punkte 0, 1, 2, 3, 4 dargestellt werden. Wir wählen für 0, 1, 2, ... die Punkte $M, M_I, M_{II}, \ldots$ und haben dann

$$\left.\begin{array}{ll} C_0=(M_I\, M_{II}\, M_{III}\, M_{IV}\, M_V)\,, & C_1=(MM_{II}\, M_{III}\, M_{IV}\, M_V)\,, \\ C_2=(MM_I\, M_{III}\, M_{IV}\, M_V)\,, & C_3=(MM_I\, M_{II}\, M_{IV}\, M_V)\,, \\ C_4=(MM_I\, M_{II}\, M_{III}\, M_V)\,, & C_5=(MM_I\, M_{II}\, M_{III}\, M_{IV}). \end{array}\right\} \quad (177)$$

Wenn wir Gleichung (158) wiederholt kovariant differentieren, so finden wir

$$C_5=(MM_I\, M_{II}\, M_{III}\, M_{IV})=(-10\,QU)^{10}\,.\,(MM_1\, M_2\, M_3\, M_4),$$

also nach (145):

$$C_5=\tfrac{2}{9}\,U^4\,(-10\,QU)^{10}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

Ferner ist nach (157), da C_5 die Gewichte 370 und 190 hat,

$$C_4=(C_5)_I=-10\,QUC_5'+(140\,QU'+100\,Q'U)\,.\,C_5=0. \quad (179)$$

Dass man auch C_3 weglassen kann, wenn man dafür die durch (133) gegebene Invariante

$$A=-\frac{73}{288}\,R^2+\frac{49}{24}\,Q'\,R-\frac{9}{8}\,Q'^2+QQ''-\frac{11}{6}\,QR'-QS \quad (180)$$

zum System rechnet, werden wir sogleich zeigen.

Es bleiben dann für das wesentliche System:

$$Q, U, V, A, C_0, C_1, C_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

Sind diese Invarianten als Funktionen des Parameters t gegeben, so ist die Regelfläche F bis auf projektive Transformationen des Raumes R_4 *bestimmt.*

Beweis. Man kennt dann die Funktionen C_i von (177), kann also M_V und damit alle höheren $M_{VI}, M_{VII}, \ldots$ linear durch die fünf Punkte $M, M_I, \ldots, M_{IV}$ ausdrücken. Dabei ist das vierdimensionale Simplex

$M\,M_I\,M_{II}\,M_{III}\,M_{IV}$ willkürlich gewählt. Aus (175) erhält man weiters durch fortgesetzte kovariante Differentiation Ausdrücke für $0_{ik}, 1_{ik}, 2_{ik}, \ldots$ in denen nur die Invarianten (181) und deren Ableitungen und nur die Punkte M bis M_{IV} vorkommen. Also liegen dann die Werte von $0_{ik}, 1_{ik}, \ldots$ für $t=0$ fest, womit auch $0_{ik}(t)$, also F gegeben ist.

Wir haben jetzt noch den Beweis dafür nach zu tragen, dass man im wesentlichen System die Invariante C_3 durch A ersetzen kann. Zu diesem Zwecke vereinfachen wir uns die Rechnung durch eine Normierung, die sich durch die Gleichungen

$$Q \equiv 1\,,\ U \equiv -\tfrac{1}{10} \quad \ldots\ldots\ldots \quad (182)$$

ausdrücken lässt. Man erreicht dies durch Wahl eines projektiv-invarianten Parameters $\tilde{t}$ und eines Faktors $\lambda(t)$ für welche

$$Q \,.\, \varphi'^{9} \,.\, \lambda^5 \equiv 1 \quad \text{und} \quad U \,.\, \varphi'^{20} \,.\, \lambda^{10} \equiv -\tfrac{1}{10}$$

wird. Es gilt dann $Q' \equiv 0$, $U' \equiv 0$ und nach (157) wird

$$K_I = K_1 = \frac{dK}{dt},$$

d. h. die kovariante Ableitung geht in die gewöhnliche über.

Wir finden bei dieser Normierung weiters, dass nach (167*a*)

$$V = -\tfrac{1}{24} R \quad \ldots\ldots\ldots \quad (183)$$

und nach (172)

$$W = -\tfrac{1}{120} R' \quad \ldots\ldots\ldots \quad (184)$$

wird. Demzufolge geht jetzt (175) über in

$$0_{ik} = -(MM_1)_{ik} \,.\, (\tfrac{25}{12} R^2 + 25 R' + \tfrac{5}{4}) + 25 R \,.\, (MM_2)_{ik} - 300 \,.\, (M_1 M_2)_{ik}. \quad (185)$$

Setzen wir

$$\omega = \tfrac{25}{12} R^2 + 25 R' + \tfrac{5}{4}$$

und schreiben (rs) statt $(M_r\,M_s)_{ik}$, so ergeben sich aus (185) die Gleichungen

$$\left.\begin{aligned}
0_{ik} &= -\,\omega\,(01) + 25 R\,(02) - 300\,(12)\\
1_{ik} &= -\,\omega'\,(01) + (25 R' - \omega)\,(02) + 25 R\,(12) + 25 R\,(03) - 300\,(13)\\
2_{ik} &= -\,\omega''\,(01) + (25 R'' - 2\omega')\,(02) + (50 R' - \omega)\,(12) + (50 R' - \omega)\,(03) +\\
&\qquad + 50 R\,(13) + 25 R\,(04) - 300\,(23) - 300\,(14)\\
3_{ik} &= -\,\omega'''\,(01) + (25 R''' - 3\omega'')\,(02) + (75 R'' - 3\omega')\,(12) +\\
&\quad + (75 R'' - 3\omega')\,(03) + (150 R' - 2\omega)\,(13) + (75 R' - \omega)\,(04) +\\
&\quad + 50 R\,(23) + 75 R\,(14) + 25 R\,(05) - 600\,(24) - 300\,(15).
\end{aligned}\right\}\quad (186)$$

Mit diesen Ausdrücken kann man $S = 0_{23,33}$ ermitteln. Eine längere Rechnung ergibt, da jetzt nach (178)

$$C_5 = \tfrac{2}{9} \cdot 10^{-4} \quad \ldots \ldots \ldots \quad (187)$$

ist:

$$S = f(R, R') + 3 \cdot 100^2 \cdot C_3, \quad \ldots \ldots \ldots \quad (188)$$

d. h. C_3 kann durch S und nach (180) also durch A ersetzt werden.

Wir bemerken noch, dass auch die Punkte $H, H_1, H_2, \ldots$ beim Aufbau eines wesentlichen Systems Verwendung finden können. So haben wir bei obiger Normierung z. B. nach (167a)

$$H = -\frac{10}{3} RM + 40\, M_1 \quad \ldots \ldots \ldots \quad (189)$$

Daraus findet man durch Differentiation Ausdrücke für $H_1, H_2, \ldots$ und kann dann die Invariante $\triangle_H$ von (88) durch $R, R', \ldots$ und die C_i von (177) ausdrücken.

Chemistry. — *The Exact Measurement of the Specific Heat of Metals at High Temperatures.* **XXXI.** *The Mean Specific Heat of Cobaltum in Connection with the Granular Size of its Crystalline Structure.* By F. M. JAEGER and A. J. ZUITHOFF.

(Communicated at the meeting of June 29, 1940.)

§ 1. The contents of this paper can be considered as a sequel to our previous publication recently issued on the properties of metallic *cobaltum* [1]). Moreover, we wish to draw attention in it to some phenomena of more general interest, because they prove to allow of a clearer insight with respect to the influence which the special coarseness of the crystalline structure of metals seemingly has on their physical properties, — more especially on the numerical values of their mean specific heats. The latter fact has previously been emphasized by us in the course of the same kind of measurements made with other metals in this laboratory [2]); and the present results evidently corroborate our earlier conclusions in a particularly striking way.

In the paper referred to, we made use of two samples of *cobaltum*: one massive sample of very pure cobaltum, consisting of *large* crystallites and furnished by Dr. DE BOER of the PHILIPS' WORKS in Eindhoven and in this paper designated as *cobaltum-A*; and a second specimen from the *Union minière du Haut Katanga* in *Brussels*, — here indicated as *cobaltum-B*, — which was electrolytically deposed and which proved to be composed of *extremely small* crystallites, which were hardly perceptible in a microscopical enlargement of 600—800 times. The two samples were very hard, sample-*B* being the hardest of the two. Whilst *cobaltum-A*, when brought into dilute hydrochloric acid in excess and heated on the waterbath, proved to dissolve *readily* within a few days, an equal weight of *cobaltum-B* treated in the same way did not or only *extremely slowly* dissolve even after several weeks, — notwithstanding the fact, that it contained somewhat more. — although very little, — cobaltous oxide and about 1 % less of metallic cobaltum than the other sample. Only after preliminary heating of *cobaltum-B* at about 1000° C. during a long time,

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1) F. M. JAEGER, E. ROSENBOHM and A. J. ZUITHOFF. Rec. d. trav. d. chim. d. Pays-Bas, **59**, 831, (1940).

2) F. M. JAEGER and E. ROSENBOHM, ibid., **53**, 456 a. f. (beryllium), (1934); F. M. JAEGER and W. A. VEENSTRA, ibid., **53**, 924, 925 a. f., (*zirconium-B* and *-C*), (1934); T. J. POPPEMA and F. M. JAEGER, ibid., **38**, 824, (1935). Conf. also: F. M. JAEGER, E. ROSENBOHM and J. A. BOTTEMA, Rec. d. trav. d. Chim. d. Pays-Bas, **52**, 61, (1933).

the solubility proved to have increased to that of the other sample and on microscopical examination it now appeared to have obtained a rather coarser texture, composed of readily discernable grains.

This strange behaviour induced us to compare its mean specific heat $\bar{c}_p$, — starting at low temperatures, — with that of *cobaltum-A*. The results of these investigations are communicated in the following sections.

§ 2. The measurements of c_p were made in the usual way, the metal being enclosed in a vacuum platinum crucible of the ordinary type. Starting at 588° C., the results of these determinations at increasing temperatures are recorded sub 1—9 in the Table I.

Evidently the whole curve thus determined is situated *beneath* the curve for sample-*A*: the $\bar{c}_p$-values at the same temperature are at 588° C. about 2 %, at 738° C., about 1,8 % and between 797° C. and 1181° C. about 0,6 % (mean values) *lower* than in the case of sample-*A*. The transition-point (1125° C.), however, has remained *un*changed; and, as the two curves have about the same course, also the *true* specific heats C_p of the two samples are practically identical. After this heating at 1182° C. the values of $\bar{c}_p$ were once more determined, now starting at 649° C., until 1296° C. was reached. This time all values of $\bar{c}_p$, up to 1000° C., *were seen practically to co-incide* with those of the curve for sample-*A*; but at temperatures surpassing 1000° C. the curve takes a course *intermediate* between that for *A* and the curve for *B* obtained by measurements 1—9;

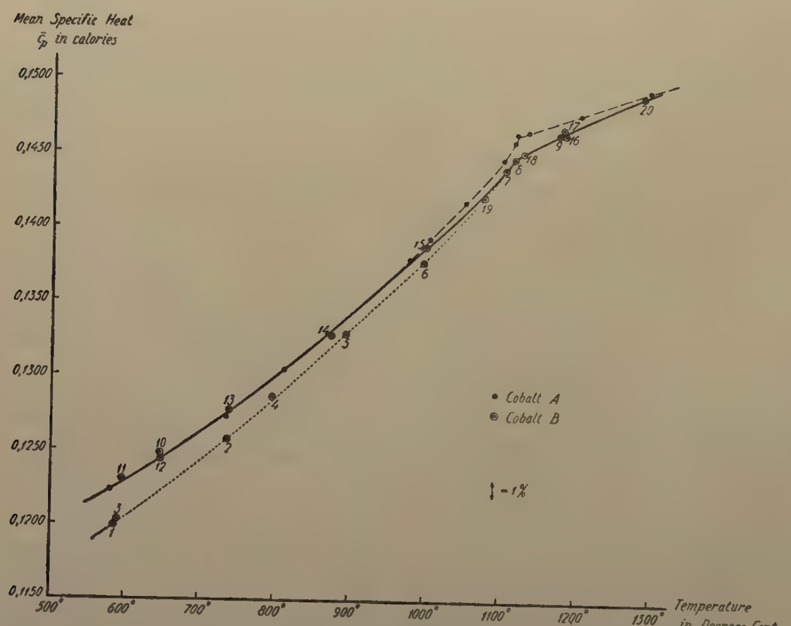

Fig. 1. *The mean specific heats of Cobaltum-A and of Cobaltum-B between 580° and 1350° C.*

TABLE I. Mean Specific Heats $\bar{c}_p$ of Cobaltum-B (Katanga) between 100° and 1400° C.

Sequence-number of Measurement:	Temperature t in Cent.:	Final temperature t' of the Calorimeter:	Increase of the temperature of the Calorimeter $\triangle t$ in mikrovolts:	Quantity of Heat Q developed by 1 Gr. between $t°$ and 25° C in cal.:	B Mean Specific Heat $\bar{c}_p$ between t and t' in cal.:	A Mean specific Heat $\bar{c}_p$ of sample-A:	Differences $(A-B)$:	Differences $\triangle$ in percent:
1	587°.7	21°.45	318.72	67.48^8	0.1199	0.1224	+0.0025	
2	738.2	22.76	419.19	89.65^6	0.1257	0.1275	0.0018	Mean: 1.8%
3	589.7	21.66^7	320.64	67.97^2	0.1204	0.1226	0 0022	
4	797.5	23.04^5	462.41	99.30^0	0.1285	0.1296	0.0011	
5	895.1	23.98	534.95	115.57^4	0.1328	0.1332	0.0004	
6	1000.1	23.08	618.19	134.13^4	0.1376	0.1383	0 0007	
7	1106.4	24.16	709.92	155.69^3	0.1440	0.1444	0.0004	Mean: 0.6%
8	1120.1	23.76	723.99	158.41^1	0.1447	0.1458	0.0011	
9	1181.2	25.50	771.12	169.16^5	0.1463	0.1472	0.0009	
10	649.1	24.14	363.02	77.77^2	0.1246	0.1243	−0.0003	
11	596.3	23.07	329.07	70.24^4	0.1230	0.1228	−0.0002	
12	648.8	22.98	362.97	77.59^6	0.1244	0.1246	+0.0002	till 1000° C
13	739.9	23.36	425.07	91.16^8	0.1277	0.1276	−0.0001	practically
14	876.8	22.42	523.73	112.96^0	0.1327	0.1322	−0.0005	0%
15	1002.2	24.75	623.07	135.67^2	0.1388	0.1386	−0.0002	
16	1184.8	24.08	774.37	169.70^0	0.1463	0.1476	+0.0013	
17	1185.4	25.65	775.64	170.27^0	0.1467	0.1478	+0.0011	Mean: 0.8%
18	1130.5	24.96	731.53	160.28^2	0.1450	0.1462	+0.0012	
19	1079.9	24.42	686.09	149.78^6	0.1420	0.1431	+0.0011	
20	1295.9	24.75	861.40	189.11^1	0.1488	0.1490	+0.0002	0.1%

the transition-point is again *not altered* and about 1125° C., the divergence of the higher and lower curves remains 0,8 %, but then gradually approaches the curve for the β-modification of sample-A, so as to *completely* coincide with it at about 1320° C. The deviation of the two curves is greatest at the CURIE-point and decreases afterwards up to 1320° C., — where it becomes zero. Evidently the heat of transformation still makes itself apparent *above* the CURIE-point.

A special controlling experiment was made, moreover, for the purpose of ascertaining that the values obtained do indeed correspond to *real* equilibria: in experiment 16 the normal time (45 minutes) of heating at a constant temperature was increased to 60 minutes, in 17 to 90 minutes; and in this way it was found that the results of experiments 16, 17 and 9, — all at 1181° — 1185° C., — were in full accordance with each other.

The data of Table I and those of Table II of our previous paper on cobaltum (loco cit.) are graphically represented in Fig. 1 besides each other.

§ 3. By these experiments it has been clearly established, that the mean specific heats $\bar{c}_p$ of *cobaltum* are found *to be so much lower, the more its composing crystallites are smaller:* a fact in plain agreement with that previously stated in the case of other metals studied in this laboratory, — where in general it was found, that the values of c_p observed were *variable with the degree of division* of the materials considered. Thus in the case of *antimony* [1]) c_p of the metal as small granules proved to be about 2 % lower than that of *antimony* in the form of a compact lump; and as well from MAGNUS and HOLTZMANN's experiments with *beryllium* [2]), as from our investigations [3]) on the same metal, it must be concluded, that c_p depends on the size of the grains constituting the metal. With *zirconium* and *beryllium* [2]) the values of $\bar{c}_p$ for powdered metals appeared *higher* than those for massive bars; but the fact, that the deviations here considered can occasionally be in the *one*, or, with other instances, in the *opposite* direction, — was also clearly stated during our studies [5]) on the *influence of prealable cold working* of such metals on the values of c_p measured. For in these processes of crushing and drawing, also a change in the *size* of the particles in the metals doubtlessly occurs to a larger or smaller extent. Another most remarkable circumstance is, that the occasionally observed "retardation"-phenomena at, or in the vicinity of, the transition-points in the massive lumps of such metals, are seen strongly to diminish and even totally to disappear, when the same materials are

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1) T. J. POPPEMA and F. M. JAEGER, loco cit.

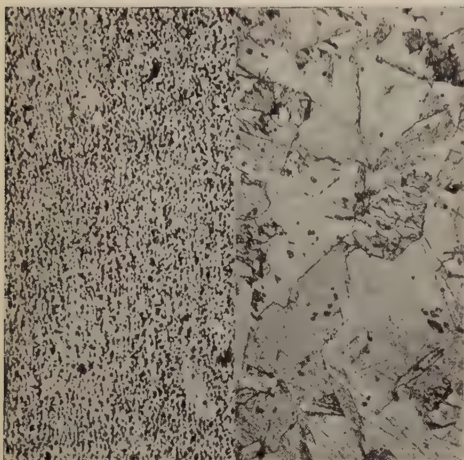
2) A. MAGNUS and H. HOLTZMANN, Ann. d. Phys., **3**, 585, (1929).

3) F. M. JAEGER and E. ROSENBOHM, Proc. Kon. Akad. v. Wetensch., Amsterdam, **37**, 67, (1934); Recueil, loco cit.

4) F. M. JAEGER and W. A. VEENSTRA, loco cit.

5) F. M. JAEGER, E. ROSENBOHM and J. A. BOTTEMA, loco cit.; conf. p. 84.

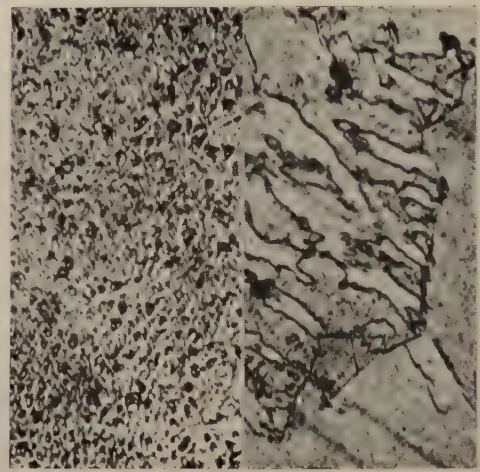
F. M. JAEGER and A. J. ZUITHOFF: THE EXACT MEASUREMENT OF THE SPECIFIC HEAT OF METALS AT HIGH TEMPERATURES. XXXI. THE MEAN SPECIFIC HEAT OF COBALTUM IN CONNECTION WITH THE GRANULAR SIZE OF ITS CRYSTALLINE STRUCTURE.



a.
Cobaltum-B

before heating, attacked with aqua regia.	after repeated heating, attacked with aqua regia (25 %)

Enlargement: 60 times.



b.
Cobaltum-B

The same as in *a*, but enlargement: 275 times.

Fig. 2. *Microphotographs of Cobaltum-B before and after heating.*

Fig. 3. *Cobaltum-A*

without preliminary heating; etched with 5 % aqua regia.

Enlargement: 7,5 times.

finely powdered, — the degree of division of the crystallites thus being considerably increased. In the case of *zirconium* it could even be demonstrated, that *sintering* of the powdered material into a coherent mass caused the hysteresis-phenomena once more to return [2]).

In the case of *cobaltum* doubtlessly the "recrystallisation" of the metal after heating at higher temperatures must be the cause of the fact, that the values of $\bar{c}_p$ of both *cobaltum-B* and *cobaltum-A* become identical again between 500° and 1000° C. But then the other fact still remains, that about 125° *below* and till 200° *above* the CURIE-point, *the original deviations persist* and that *they reach a maximum value at this transition-point itself*. It must be evident, that here the influence of the *heat of transition* at this point must be the cause of the fact, that curve *A* still remains higher situated than curve-*B*. It is this heat of transition corresponding to the transformation at the CURIE-point, which, — making itself already perceptible some 125° below this temperature, — appears to be scattered over a considerable range of temperatures before the CURIE-point is reached, — just as in the case of pure *iron* [6]). This particular interpretation in the case of the finely-crystallized *cobaltum-B* seems to be justified to a high degree, if it may only be supposed, that the transition-heat at 1125° C. now was *appreciably smaller,* than in the case of the coarse-crystalline *cobaltum-A*. The situation of the transition-temperature itself, however, is seen *not* to be altered by this fact.

§ 4. Finally, as a result of the microscopical examinations, we here insert fig. 2, which shows a microphotograph *a* of the original preparation at room-temperature and after heating to 1200° C.; and a micro-photograph *b* of the same samples at an enlargement of 275 times. The increase in size of the crystallites by the gradually achieved recrystallisation is quite evident. The *heated* sample proved, moreover, now to have become readily soluble in dilute hydrochloric acid. For the purpose of comparison we add (in fig. 3) a microphotograph of *cobaltum-A,* which was *not* previously heated; here an enlargement was used of only 7—8 times.

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[6]) F. M. JAEGER, E. ROSENBOHM and A. J. ZUITHOFF, Rec. d. trav. d. chim. d. Pays-Bas, **57**, 1322, (1938).

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Geology. — *New data on the smaller islands North of Venezuela.* By L. RUTTEN.

(Communicated at the meeting of June 29, 1940.)

In 1931 I reviewed (4) the geological literature of the Venezuelan islands between Bonaire and Trinidad and gave some data on the base of the study of rock-samples, collected by R. LUDWIG and by P. R. LOWE. In 1936 Mr. P. WAGENAAR HUMMELINCK visited these islands on a biological voyage; he made at the same time collections of rocks which he kindly gave to my institute. The following additional data on the geology of the islands are the result of the study of these collections.

Los Frailes. Nothing seems to have been published on these small cliffs which lie quite near to the northeast-coast of Margarita [1]). There are samples from Puerto Real and from La Peche.

No. **156**, Puerto Real, SW-slope of Morro Grande, alt. 40 m, is a diabasic diorite. The main components are albitized, idiomorphic plagioclase and pyroxene, with ophitic texture; large spots of chlorite with some epidote probably originated from biotite. Some green hornblende, octahedral ore and many very slender needles of apatite; moreover traces of micropegmatite, occurring as interstitial material between the main components.

No. **157** and **158**, same locality, sampled from isolated pieces of rock; are diabases. The first is very coarse, the second is fine-grained and fluidal; in both, the plagioclases are more or less albitized.

No. **159**, found at the top of Morro Grande in large blocks, is a coarse quartz-epidote-chlorite rock. It contains octahedral limonitized ore and slender needles of apatite. Both epidote and chlorite may occur in spherulites. It is very probably a postmagmatic rock.

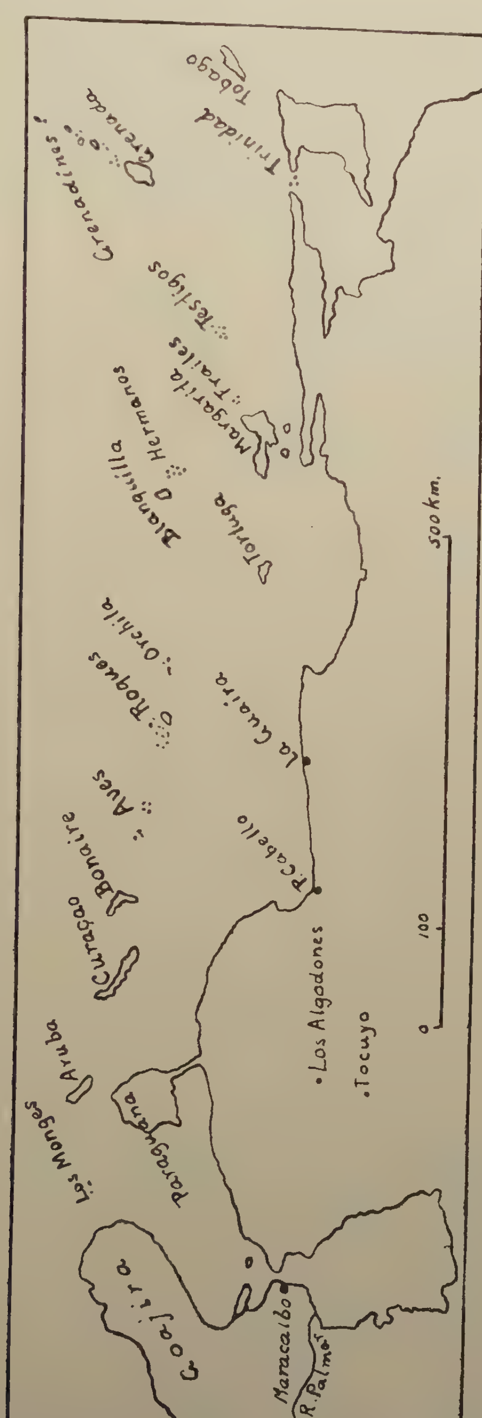
No. **160** from the SW part of La Peche is probably a quartzdioriteporphyrite which, however, has some characteristics of a diabase. It contains rare phenocrysts of decalcified plagioclase partly filled with epidote, and a groundmass with an ophitic texture, consisting of laths of albite and small crystals of a green biotite which partly have changed into spherulitical chlorite. There is some interstitial material of quartz and micro-albite-pegmatite.

No. **161**, same locality, is a green rock, consisting almost entirely of uralite and albite, with, locally, some prehnite. It is probably a metamorphic porphyrite-tuff. According to Mr. HUMMELINCK's field-notes sample **160** occurs as a "lightcoloured ledge" in **161**.

No. **162** is, according to HUMMELINCK, the contact-zone between **160** and **161**. It is a dirty-green rock, partly coarse-grained, partly fine-grained. The coarse-grained part is

[1]) In the academical thesis of Mr. HUMMELINCK (1*a*) published after the communication of this paper, some notes (p. 16) and a sketchmap (p. 44) are to be found. The latter shows that the islands are arranged "en échelon" with a WNW-strike.

a metamorphosed porphyrite-breccia, each grain being a piece of porphyrite, whilst the intermediate substance consists mainly of uralite. The fine-grained part is a metamorphic porphyrite-tuff, containing equally much uralite and a veinlet of prehnite.



No. **163**, from the N-hill of La Peche is a coarse uralite-diabase with large crystals of labradorite and uralitic hornblende, which often includes rests of pyroxene. There are many skeleton-crystals of ore, slender apatite needles and a vein of prehnite.

No. **164**, from the basal part of the S-hill of La Peche, is a strongly uralitized porphyrite-tuff: it is a quartz-uralite-albite rock with remains of porphyritic texture.

No. **165**, from the depression between the N- and the S-hill of La Peche, is equally a metamorphic, uralitized, porphyritic tuff; it consists of a fibrous fabric of uralite and albite with, locally, remains of plagioclase-phenocrysts, and with an undeniable piece of porphyrite which, although being metamorphosed, shows still clearly the texture of a groundmass. It contains a vein of prehnite.

There is an enormous difference between the magmatic rocks of Los Frailes which have not suffered from crushing forces and the schistose rocks of the near Margarita (5) which have suffered strongly from crushing. On the other hand the rocks of Los Frailes fit in very well with those of the island-row Aruba-Los Testigos, the numbers **157**, **158**, **161**, **162**, **163**, **164** and **165** resembling the rocks of the cretaceous, volcanic basement of some of these islands, and more especially the contactmetamorphic tuffs and diabases from Aruba (7). It is not certain whether the samples **156**, **159** and **160** belong to the volcanic-basement-series or to the younger group of intruding rocks, so well known from Aruba, but also from other islands. They have on the one side "dioritic relations", on the other side "diabasic relations". We suppose that, in any case, an intrusive rock must occur in the underground, near to Los Frailes, given the strong uralitization of some porphyritic tuffs and breccias.

Los Testigos. I could state in 1931 (l.c.) the presence of hornblende-granodiorite and of spessartite on these islands. Mr. HUMMELINCK took samples on Morro de Iguana, Chiwu, Angoletta, Tamarindo (Testigo Grande) and Isla de Conejo (see 1*a*, p. 15, 44). They enlarge greatly our knowledge of this group of cliffs.

No. **138**, top of Morro de Iguana, is related to the granodiorite, described in 1931. It differs from it by its femic minerals which are chlorite-epidote (ex: biotite) and pyroxene with a marginal zone of amphibole; the leucocratic components in both rocks are the same.

No. **139**, from an elevated place at the E. coast of Morro de Iguana, differs from the foregoing by the presence of more femic components (augite, bronzite, biotite, with skeleton crystals of ore). The leucocratic minerals are idiomorphic, finely twinned, zonally built andesines with more acid margins, and a filling-mass of rather coarse orthoclase and quartz. The rock may be called a biotite-pyroxene-quartzmonzonite.

No. **140**, sampled from isolated fragments of rock on Morro de Iguana is a quartz-chlorite-epidote-rock with crystals of limonitized magnetite; it is very probably a postmagmatic rock.

No. **141**, from Chiwu, a cliff, measuring about 300 × 80 m, is again a granodiorite. It contains large, porphyritic plagioclases, passing gradually into smaller ones, all idiomorphic, finely twinned and zonal. The femic minerals, probably biotite, have entirely

been changed into masses of chlorite and carbonate. There is an abundant filling mass of coarsely crystalline orthoclase and quartz.

No. **142** from Chiwu is a, probably postmagmatical, quartz-epidote-zoisite rock.

No. **143** and **145**, from the small island of Angoletta (it measures, according to Mr. HUMMELINCK, only 80 × 30 m) are uralitic quartzdioriteporphyrites. They contain phenocrysts of plagioclase and phenocryst-like spots of uralite. In both rocks the texture of the groundmass is granitic-isodiametric, and it consists of plagioclase, quartz and uralite.

No. **144** is a quartzitic rock, probably a sample from a quartz-vein. It occurs as a dike in no. **143**.

No. **146** from top of Morro Grande, Tamarindo, is a granodiorite, resembling **138**.

No. **147** which occurs as a white dike of 20 cm within the former rock, is a plagiaplite, consisting of an aplitic intergrowth of quartz and dusty, acid plagioclase (n_γ: 1.540); it contains an extremely thin veinlet of epidote.

No. **148**, from Playa Guzman, on the E. side of Morro Grande, Tamarindo, is again a granodiorite, resembling **138** and **146**, but with only a few femic components (hornblende).

No. **149**, from Tamarindo, found in isolated blocks, is a quartz-sand, cemented by phosphorite.

No. **150**, from NW-edge of Isla de Conejo, is a porphyrite with fine phenocrysts and splinters of plagioclase and with some chlorite-spots (?ex femic phenocrysts); the groundmass is very fine-grained and fluidal. The slide contains some inclusions of coarse-grained porphyrite.

No. **151**, from the same locality, is a totally epidotized and silicified porphyrite: the well-recognizable phenocrysts of plagioclase have been replaced entirely by epidote-aggregates; the groundmass, of which the fluidal texture is still visible, consists of epidote and quartz.

No. **152**, from the same locality, is a phosphorite with splinters of quartz.

No. **153**, from top of Isla de Conejo, is a porphyrite, strongly resembling **150**; so is also **154**, from the S-coast of Isla de Conejo, in which some chlorite-spots may be recognized by their form as having originated from amphibole crystals.

No. **155**, from the same locality, is again an epidotized and silicified porphyrite, entirely comparable with **151**.

The rocks of Morro de Iguana, Chiwu, Tamarindo are so alike the rocks of the batholithic sequence of Aruba (7) that they might have been collected on that island; the uralitic quartzdioriteporphyrite from Angoletta with its quartz veins belongs very probably to the same sequence. The rocks from Isla de Conejo, on the other hand, are comparable with the rocks of the volcanic basement of different islands, and there is no reason, why they should not belong to it. The Testigos islands are, as the Frailes, arranged "en échelon" with NW-strike (1*a*); Isla del Conejo is clearly separated from the rest of the group.

Los Hermanos (1a, p. 17, 18, 44). I could describe in 1931 (l.c.) a hornblendegabbro, sampled by P. R. LOWE, from these islands. The character of the rocks, collected by Mr. HUMMELINCK, is not so clear as that of the rocks from Los Frailes and Los Testigos.

There are four rocks which, although not typical, fit in rather well with the batholithic rocks in Aruba etc.

No. **254**, from N. slope of Morro Fondeadero, is a hornblendegabbro with hooibergite-habit. The very basic plagioclases (labrador-bytownite) are somewhat zonal; the amphibole is normal, green hornblende.

No. **256**, from the NW-side of Morro Fondeadere at sealevel, is quartzhornblendediorite with much apatite in coarse prisms. The partly idiomorphic plagioclases are somewhat zonal; they have the composition of andesine; there is a few quartz, occurring as filling-mass.

No. **258**, from N-top of Morro Pando, is an atypical quartzdiorite. The chief components are andesine and quartz without distinct crystallization-sequence, the quartz being, however, more clearly xenomorphic than the feldspar. The femic elements are hornblende, somewhat fibrous and very light-coloured, and small biotite-crystals. The quartz shows feeble undulatory extinction.

No. **261**, equally from N-top of Morro Pando, is a quartz-biotite-diorite with acid plagioclase (n : 1.540), sometimes strongly epidotized, quartz and streaks of biotite.

Four other rocks may belong to the batholithic sequence; they might, however, also belong to an older basement.

No. **252**, from top of Morro Fondeadero, is a coarse amphibolite; the plagioclases are labrador-andesine. At one side in the slide the amphibolite has been covered by phosphorite.

No. **257**, from NW-side of Morro Fondeadero, at sealevel (same locality as **256**) is an amphibolite with pyroxene.

Nos. **259** and **260** from N-top of Morro Pando, are heavy, black-and-white-spotted rocks with basic plagioclases (labrador-bytownite), green hornblende, some quartz and magnetite. They may be related with the hooibergites of Aruba, but the ill-developed crystallization-sequence and the strongly basic plagioclases make it questionable whether they belong to them.

Blanquilla. The only rock, known until now from Blanquilla, is a biotitegranodiorite (4). Mr. HUMMELINCK took samples at Valuchu, in the SE-part and at Puerto Llaque, in the SW-part of the island. The samples from Valuchu are partly young limestones, partly corals, of which Prof. GERTH kindly identified Favia fragum Esp., Porites astreoides Lam., Acropora muricata L., Orbicella acropora (L.), and Platygyra (Maeandra) viridis (Le Sueur), partly quaternary molluscs, which are now studied by Miss T. VAN BENTHEM JUTTING, partly quartzdioritic rocks.

Nos. **262**, **269**, **275** are quartzdiorites which differ from the earlier described granodiorite chiefly by the absence (or extreme scarceness) of orthoclase. The biotite has been partly chloritized; the idiomorphic, zonal and finely twinned plagioclases are oligoclase-

andesine to andesine; the quartz which shows undulatory extinction, occurs as filling mass, together with some albite.

No. **270** is an aplitic quartzdiorite with very scarce chloritized biotite.

Nos. **263—268** are young limestones, all containing clastic material of quartzdiorites and not containing any other clastic material; remains of organisms are rare; **270** has been partly phosphatized.

Nos. **271**, **274** are young limestones, full of organic structures.

No. **276** is a young conglomerate with calcitic cement; the grains of the conglomerate are exclusively of quartzdioritic nature; the cement contains very fine Amphiroa's.

The samples of Puerto Llaque are for the most part granitic and quartzdioritic rocks; one sample is a conglomerate.

No. **277**, comparable with **276**, is a conglomerate of quartzdioritic material with calcitic cement, wherein Amphiroa and remains of Lamellibranchiata and Echinodermata.

Nos. 278 and **281** are biotitegranodiorites; the biotite has been partly chloritized; the plagioclases occur in two generations; the slides present a filling-mass of quartz, orthoclase and myrmekite; accessories are titanite, apatite and zircon. The rocks have suffered from crushing: the quartzes show rather strong undulatory extinction.

No. **279** is a typical quartz-hornblende-biotite-diorite.

No. **280** is a granite-aplite with some large sub-idiomorphic crystals of oligoclase, with large perthitic orthoclases and with abundant, undulatory extinguishing quartzes. Part of the slide shows a micropegmatitic intergrowth of quartz and orthoclase.

No. **282** is a crushed aplite, containing some acid plagioclases and a xenomorphic intergrowth of microcline, microclineperthite, micropegmatite and quartz.

It will be clear from the foregoing that the basement of Blanquilla exists of rocks which are quite comparable with those of the batholith of Aruba: all the rocks of the basement are typical representants of this batholith, and even the clastic material in different young conglomerates and limestones belongs to the same sequence.

Orchila. Two years ago M. ROST (3) has published a geological map of Orchila, showing that the island has a basement of crystalline schists with some granitic-dioritic rocks in the W-part of the island. Different masses of "basalt" have been indicated in the centre of the island; the greatest part of the island is covered by young coral-limestones. Mr. HUMMELINCK has taken samples in the W-part of the island, but I do not think that their detailed description is of any worth, as certainly the collections of Mr. ROST which have not yet been studied will prove to be of more importance. I should only like to indicate that the schists of Orchila which have suffered very strongly from crushing, present a great relationship to those of Margarita, that mylonitized granitic aplites

from Orchila seem to be equally related to those of Margarita, and that young limestones contain very fine Amphiroa's.

Los Roques. I concluded in 1931 that the rocks of Gran Roque are closely related to the batholithic rocks of Aruba. Since then two descriptions of the island have appeared. S. E. AGUERREVERE and V. M. LÓPEZ (1) have given a detailed description with a fine geological map; M. ROST has equally given a description with a geological sketchmap (3). In his description, ROST presents a "semi-magmatical" theory on the genesis of the phosphorites of the island, which seems to me to be absolutely erroneous; it is, however, here not the place to criticize it. Mr. HUMMELINCK has sampled on Gran Roque, at the side of phosphatic rocks, many magmatic rocks which confirm absolutely the views held by me in 1931. It would be unreasonable to describe the whole collection in detail; the following is an enumeration of the types of rocks in the collection of Mr. HUMMELINCK: hornblendegabbro, uralitic gabbro, uralitic gabbro-diabase, quartzbiotitediorite, quartzamphiboledioriteporphyrite, biotitegranodiorite, biotitegranite, strongly crushed plagiaplite, quartzaplitepegmatite, granitemicropegmatite and amphibolite. With the exception of the amphibolites the samples fit in very well with the batholithic rocks of Aruba; the fine-grained amphibolites are comparable with the amphibolites that have been met-with in Aruba in the contact-zone around the batholithe (7).

On low islands of the Los Roques-group Mr. HUMMELINCK has collected some young calcareous rocks: a sandstone, consisting entirely of rounded fragments of calcite on Isla Larga (Cayo Grande) and fine-grained, partly organic limestone-breccias on Cayo de Agua.

Los Aves. On Ave de Barlovento Mr. HUMMELINCK sampled two porous limestones with organic remains, among which Amphiroa.

Tortuga. SIEVERS (6) has described Tortuga as a low, calcareous island.

Mr. HUMMELINCK took samples (nos. **363—369**) of the limestones; moreover he collected molluscs, which are studied by Miss. T. VAN BENTHEM JUTTING and corals, of which Prof. H. GERTH kindly identified the following species: Pocillopora crassoramosa Dunc., Siderastrea siderea Ell. a. Soll., Acropora muricata (L.), Orbicella acropora (L.), Madracis decactis (Lym.), Colpophyllia gyrosa Edw. a. H. and Eusmilia sp. The limestones are all very porous rocks with mostly abundant grains of quartz. They contain Amphiroa and Amphistegina at the side of ill-preserved other organic remains.

Centinela (75 km W. of Tortuga). A sample of a very fine-grained siliceous rock (nos. **361**, **362**) with veinlets of quartz and with larger ones of phosphate has been sampled on this cliff. The rock is probably a (?cretaceous) chert.

Summary.

1. The study of the new collection of rocks from the islands between Los Aves and Los Testigos confirms the conclusions, at which I arrived in 1931.

2. These conclusions are extended in so far as: a. the small group of Los Frailes has been proved to belong equally to this row, b. at the side of batholithic rocks there have been found also rocks of the volcanic basement, viz. on Los Frailes and Los Testigos.

3. The metamorphic basement of Orchila presents great resemblance with the basement of Margarita.

4. The young, quaternary, capping limestones in this group are everywhere characterized by the presence of Amphiroa which occurs equally in great abundance in quaternary limestones of Curaçao (2).

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Utrecht, June 1940.

Geology. — *On the Geology of Margarita, Cubagua and Coche (Venezuela).* By L. RUTTEN.

(Communicated at the meeting of June 29, 1940.)

Mr. P. WAGENAAR HUMMELINCK, of the Utrecht University, made in 1936 a voyage to the North Coast of Venezuela and to the islands North of Venezuela. His purpose was in the first place to make zoological collections; at the side of this he did also geological observations and collected rocks, which he put kindly at my disposal. The following is the result of the study of these rock-collections as far as regards the large island of Margarita and the small islands Cubagua and Coche.

The following data on these islands are to be found in the literature. DAUXION DE LAVAYSSE describes Margarita as being very dry and unfertil; the Macanao mountains are said to consist of micaschists (2). WALL (11) indicates the existence of his "Caribbean System" of metamorphic rocks and of younger sandstones of unknown age with a maximal dip of 35°; these sandstones crop out between Porlamar and Pampatar. W. SIEVERS (10) mentions, on the base of the diary and the collections of R. LUDWIG, the occurrence of gneiss-micaschists, graphite schists and phyllites, all considered as archaean rocks, of tertiary limestone in the neighbourhood of San Juan Griego, and of serpentine. C. MAURY (5), without giving details, says that there occur probably sediments of Midway-age on the island. R. A. LIDDLE (4), in his well-known book on Venezuela and Trinidad, is rather explicit on Margarita. He gives (p. 72) a N—S section, wherein he indicates: 1? Palaeozoic or Basal Cretaceous (schists and metamorphic limestone, shale and sandstone), 2. Basal Cretaceous? (partly altered limestone and sandstone capping mountains), 3. ?Lower Cretaceous (sandstone, shale and limestone), 6. granite (mica-hornblende granite, schistose at edges of mass., magnesite deposits at granite and schist contact). (1) and (2) are separated from (3) by a fault near Porlamar. LIDDLE's text (p. 73—74) is, unhappily, not in agreement with his section. In his section he calls the sandstone-shale formation of Porlamar "Lower Cretaceous", in his text "Upper Cretaceous". Moreover he states that it is impossible to differentiate the schistose rocks of the island into Precretaceous and Cretaceous, and he does not at all prove that part of the schists is of cretaceous age. He only says that he regards the marbles of the island as metamorphosed "El Cantil limestone", and that the "shiny mica schists which comprise the greater part of the island ...... are occasionally capped by a few meters of this marmolized limestone". HESS (3) mentions the occurrence of serpentines and says that in the

West Indies "the serpentines intrude Upper Cretaceous rocks for the most part" (p. 85). In recent time a special paper on Margarita has been published by P. I. AGUERREVERE (1); it is accompanied by a geological map. He states that metamorphic rocks (micaschists, gneiss, serpentine and crystalline limestone) are predominant; of these, the marbles are thought to be the youngest and of cretaceous age. A coarse granite intrudes the metamorphic rocks near Paraguachi; the existence of basic eruptive rocks is possible. A strongly folded formation, regarded as eocene, is found in the neighbourhood of Porlamar and Pampatar. A rock at the W.hill of the "Tetas de Maria Guevar" is considered to be a black lava; the description, however, is not very convincing: "una lava de color casi negro, excepto sus cristales de cuarzo que son blancos o sin color" (p. 402). Data about miocene and pliocene strata are very vague.

Cubagua and Coche are low islands (LIDDLE); SIEVERS supposes that Cubagua consists of archaean rocks; AGUERREVERE mentions from East Coche metamorphic rocks and variegated shales, capped by a ?pliocene conglomerate, containing pebbles of quartz and marble. These pebbles indicate that, formerly, Coche must have belonged to a greater land-area [1]).

The rocks of the three islands in the collection HUMMELINCK belong to:

1. The metamorphic basement of Margarita and Coche,
2. The Eocene of Margarita,
3. The young "capping" formations (?Pleistocene) of the three islands.

They will be discussed in this order. For localities, see the accompanying map.

Basement of Margarita. A. G n e i s s e s. no. **5**, **6**, **7**, **8**, **10**, **13**, **25**, **36**, **194**, **195**, **240**, **371**, **372**, **373**, **375**. (No. **36** from an isolated rock-fragment; the others all outcropping). The gneisses have been found exclusively in the eastern part of the island; it must, however, been taken in mind that the W.part has been very poorly sampled. In the literature gneisses have been only mentioned by AGUERREVERE (1); microscopical descriptions are absolutely lacking.

The rocks **5** and **6** are sericite-albitegneisses [2]). Both contain porphyroblasts of albite (often with inclusions of sericite), and a mosaic of small quartzes and albites with some sericite, somewhat differentiated into layers. The twinning lamellae of the large albites are bent and sometimes broken. Both rocks have been exposed to crushing forces after their gneissification. The samples **36**, **371** and **375** are equally sericite- or muscovite-albitegneisses; **194** and **195** differ from the foregoing by strong

[1]) Some geological data on the three islands are to be found in the academical thesis of Mr. HUMMELINCK (3a, p. 16, 17, 45, 46, 47) which was published after the communication of this paper.

[2]) In the following I shall call "albite" the plagioclases with a refraction index of less than 1.540. These acid plagioclases, which are very common in the Margarita-rocks include albite s. str. and albite-oligoclase.

mylonitization. Quartz and albite have been crushed along more or less parallel zones, and the remaining larger crystals show strong undulatory extinction; **7** and **10** are analogous.

372 and **373** are albitegneisses with garnet; **372** contains moreover muscovite, chlorite, some biotite and tourmaline, **373** contains sericite and zoisite. **240** is a fine-grained, schistous albitegneiss with chlorite and zoisite.

Sample **8** has the habit of a coarse, schistose muscovitegneiss. It contains: a. some large, perthitic orthoclases, b. rather many large albites, partly with inclusions of sericite and idiomorphic epidote, c. small crystalls of (a) and (b) which probably originated — by crushing — from larger ones, d. large spots of strongly undulatory and cataclastic quartz, e. streaks with epidote, muscovite and some biotite, f. some large titanites, g. some fibrous sillimannite. Clearly, there is a large difference between this sample and the foregoing gneisses. Possibly this difference is related with the fact that no. **8** has been found at the same locality as a granitic rock.

No. **13** has a character of its own. The leucocratic minerals — quartz and albite — form a minority; the bulk of the finely schistous rock being composed of muscovite, epidote (mostly idiomorphic) and hornblende (in idiomorphic prisms; pleochroism yellow — greenblue). Accessories are rutile, chlorite, garnet and ore. This rock is from Puerto Manzanillo, where very basic rocks predominate.

We see that most of the gneisses are albite-gneisses with no great variation; only two samples are very different from the rest: the albite-orthoclase-gneiss no. 8 and the melanocratic albite-gneiss no. 13. Many rocks present signs of strong, crushing dynamometamorphosis posterior to the gneissification.

B. Micaschists. Samples: **2**, **45**, **54**, **55**, **56**, **59**, **69**, **70**, **72**, **73**, **74**, **75**, **79**, **80**, **81**, **82**, **97**, **187**, **192**, **193**, **235**, **236**, **244**, **247**, **248**, **251**, **370**, **374**, **377** (no. **72** and **244** from isolated rock fragments; the other ones from outcropping rock). The micaschists have been found at numerous localities throughout the island. They have formerly been mentioned by DAUXION DE LAVAYSSE (2), WALL (11), SIEVERS (10), LIDDLE (4) and AGUERREVERE (1); descriptions of the rocks are entirely lacking.

Most rocks are muscovite- and sericite-quartz-schists; biotite is extremely rare. Generally the schistosity is well developed in consequence of the concentration of mica and quartz in alternating layers. The quartzes are often interlocked. In many cases (f.i. **74**, **244**) the quartzes are cataclastic at their periphery, indicating that the rocks have suffered from crushing after their metamorphosis. Rarely, some plagioclases are found (**74**, **248**), the rocks passing then into gneissic micaschists. The following accessoria are found.

Chlorite occurs frequently (f.i. **2**, **45**, **75**, **192**, **248**); it may even be present in equal quantity as mica.

Many samples are rich in graphite (**2**, **45**, **72**, **82**, **97**, **187**, **235**, **247**,

251); they become very dark and even black, f.i. at the Tetas de Maria Guevar.

Garnet (**54**, **56**, **59**, **192**, **193**, **244**, **247**, **248**, **370**) is found frequently, mostly in isodiametric, xenomorphic grains with sieve-structure. All the garnet-containing rocks are relatively coarse.

Tourmaline (pleochroism: lightbrown — colourless) in idiomorphic prisms occurs in the samples **187**, **192**, **236**, **248**.

Rutile has been found in **79**, **248**, **251**; Zircon and Pyrite are common.

At the side of true micaschists there occur rarely quartzschists (**55**), whilst **97** is a quartz-graphite schist, in which the graphite has been concentrated in thin layers; the rock gives the impression of having originated from a chert.

There is one sample which deserves special description (**192**). It is a quartz-muscovite-schist with garnet and tourmaline, and with many crystals of a blue mineral with the following characteristics. Refraction high; pleochroism lightyellow-blue; one good cleavage; optical angle small and optical character positive, which can be stated in sections parallel to the cleavage; angle of extinction with regard to the cleavage maxim. 20°. The mineral is chloritoid, agreeing absolutely with chloritoids from Piemont, Wallis and Rhode Island.

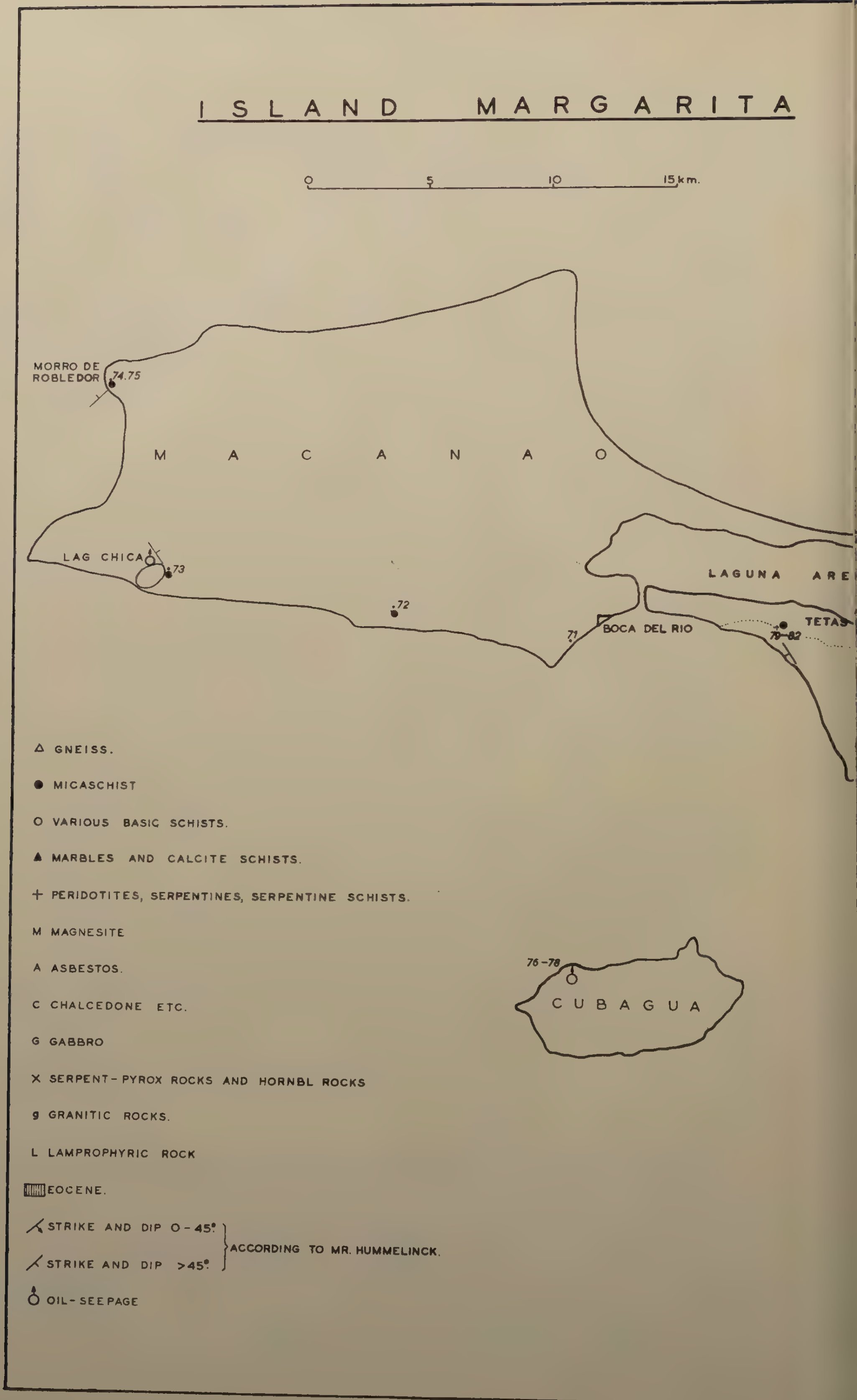
C. Various Basic Schists. Samples: **37**, **38**, **39**, **40**, **41**, **49**, **50**, **51**, **52**, **92**. No data in the literature.

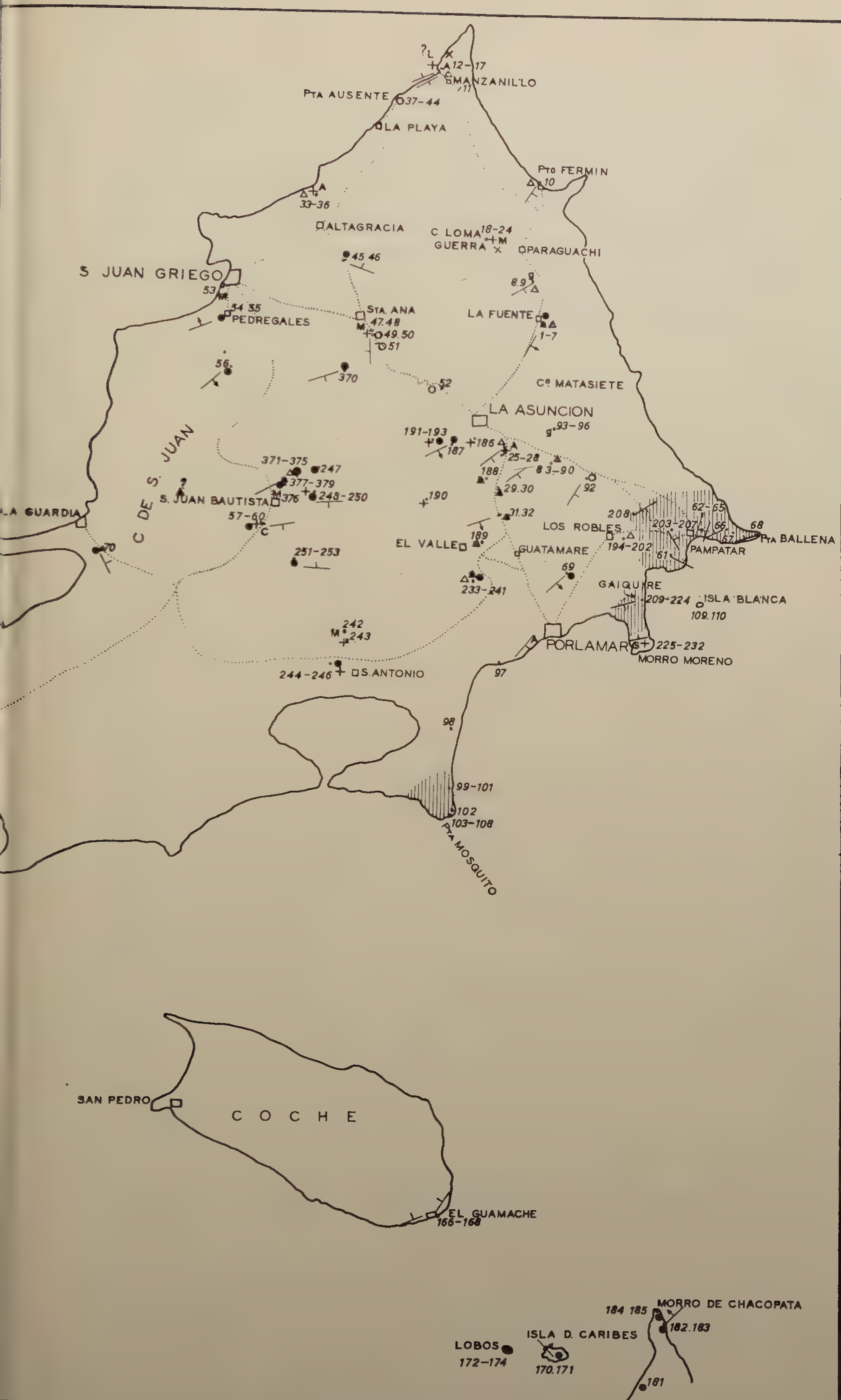
The samples **37**—**41** are all from Punta Ausente, where no other basement-rocks have been collected (**42**—**44** are subrecent limestones). No. **37** is an amphibole-eclogite with fine prisms of amphibole (yellow-greenblue), garnet (with inclusions of quartz), grains and idiomorphic crystals of epidote-zoisite, many plates of muscovite, grains of rutile, and with some quartz as matrix. No. **38** is a subschistose green rock with predominating hornblende and epidote, less muscovite, and with accessory quartz, chlorite and rutile; it is an amphibole-epidote-zoisite-rock. Nos. **39** and **40** are poorly schistose; they are composed of large, clear albites with many inclusions of blue-green hornblende and epidote-zoisite, and of large crystals of hornblende and epidote. Rutile is also present. The rocks are epidote-amphibole-albite-rocks. No. **41** has the same habit as the foregoing; it is, however, an epidote-muscovite-albite-rock. The samples **37**—**41** are very similar to inclusions, which are found in the serpentine-massives of Santa Clara province, Cuba (8a).

The samples **49**—**52** have been found on the road from Sta. Ana to La Asunción; they are fine-grained, schistose amphibolites, containing amphibole, albite and epidote-zoisite; in **51** and **52** there is, moreover, some mica.

The last sample (**92**) is an actinolite-schist with accessory zoisite and plagioclase.

An albite-chlorite-schist (**25**) may be regarded as an intermediate between the micaschists and the basic schists.


ISLAND MARGARITA
0 5 10 15 km.
MORRO DE ROBLEDOR
74.75
MACANAO
LAG CHICA
73
72
71
BOCA DEL RIO
LAGUNA ARE
TETAS
79-82
76-78
CUBAGUA
GNEISS.
MICASCHIST
VARIOUS BASIC SCHISTS.
MARBLES AND CALCITE SCHISTS.
PERIDOTITES, SERPENTINES, SERPENTINE SCHISTS.
M MAGNESITE
A ASBESTOS.
C CHALCEDONE ETC.
G GABBRO
X SERPENT-PYROX ROCKS AND HORNBL ROCKS
g GRANITIC ROCKS.
L LAMPROPHYRIC ROCK
EOCENE.
STRIKE AND DIP 0-45°
STRIKE AND DIP >45°
ACCORDING TO MR. HUMMELINCK.
OIL-SEEPAGE



MANZANILLO
PTA AUSENTE
LA PLAYA
PTO FERMIN
ALTAGRACIA
C LOMA GUERRA
PARAGUACHI
S JUAN GRIEGO
PEDREGALES
STA. ANA
LA FUENTE
Cº MATASIETE
LA ASUNCION
C DE S. JUAN
S. JUAN BAUTISTA
LA GUARDIA
LOS ROBLES
PAMPATAR
PTA BALLENA
EL VALLE
GUATAMARE
GAIQUIRE
ISLA BLANCA
PORLAMAR
MORRO MORENO
S. ANTONIO
PTA MOSQUITO
SAN PEDRO
COCHE
EL GUAMACHE
MORRO DE CHACOPATA
ISLA D. CARIBES
LOBOS

D. Crystalline Limestones.

Samples **1**, **3**, **4**, alternating with micaschists and gneiss, and, according to the field-notes of HUMMELINCK, also with quartzites.

Samples **188**, **29**, **30**, **31**, **32**, **189**, **83—90**, **233**, **234**, **237**, **241** apparently belong to a broad zone with ENE-strike and S-dip. According to the field-notes of HUMMELINCK the marbles are forming entire hills; at **31**, **32** schists, marbles and quartzites are alternating; at the locality **83—90** there is equally an alternation of marbles and schists; from **188** schists with strata of marble are mentioned; in the marble of **241** is a large cave (Cueba del Piache, 3a, p. 54).

Samples **378**, **379**, from large, isolated blocks. The people of San Juan told Mr. HUMMELINCK that much marble occurs in the Cerros de San Juan.

Evidently the marbles have been sampled in three districts: the area S. and E. of La Asunción, the surroundings of S. Juan Bautista, and the area around La Fuente [1]).

Crystalline limestones from Margarita have been mentioned by LIDDLE (4) and by AGUERREVERE (1). As is to be seen from the foregoing and on the map the marbles are often alternating with other metamorphic rocks, whilst the whole zone with crystalline limestone S. of La Asunción apparently lies between other schists. It is certainly not true that the marbles are "capping mountains" (LIDDLE), nor has it been proved that they are the youngest element of the basement-rocks (AGUERREVERE). As far as can be judged they form normal intercalations within the schist-series.

The rocks under consideration are sometimes without schistosity, white or grey-white (**29**, **30**, **84**, **85**, **87**, **88**, **90**, **189**, **233**, **234**, **237**, **241**, **378**, **379**) or they are schistose, sometimes hardly recognizable as calcite-schists, and with colours: white, green-white, grey, black-grey (**1**, **3**, **4**, **31**, **83**, **86**, **89**, **188**). In the first group calcite is always strongly predominating; in the second group the accessories may equal or even surpass the calcite. In some cases there are hardly any accessoria (**237**, **378**, **379**). Where present, the accessories are: quartz, albite, sericite-muscovite; more rarely epidote-zoisite, chlorite, biotite; very rarely tourmaline (**3**, **84**), garnet (**86**), titanite in grains (**4**), pyrite (**84**), and graphite-dust. Tourmaline and epidote may be idiomorphic; quartz occurs mostly as "drops"; it may, however, also occur in thin layers with interlocked crystals.

In the schistose samples the calcite-crystals are always elongated in the planes of the schistosity; in the true marbles they may be isodiametric.

As in the case of the gneisses and the micaschists, it can sometimes be observed that the marbles have suffered from crushing forces after their

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[1]) According to the map of AGUERREVERE marbles are to be found also in the W. part of the island.

crystallization, a cataclastic zone having developed between the large grains of calcite.

E. Peridotites, Serpentines, Asbestos, Gabbros, Magnesite etc.

Samples: Serpentinized peridotites **18**, **191**, **225**, **227**, **230**; Serpentines and Serpentine-schists, often with many accessories **16**, **20**, **21**, **26**, **28**, **33**, **34**, **48**, **58**, **60**, **186**, **190**, **229**, **232**, **243**, **246**, **249**, **250**; Magnesite **19**, **47**, **53**, **242**, **376**; Asbestos **14**, **15**, **27**, **35**; Serpentine-pyroxene-rock **17**, **22**, **23**; Hornblende-rock **24**; Gabbro **226**, **228**, **231**; Chalcedone etc. **57**.

According to the field-notes of Mr. HUMMELINCK the samples **14—17** have been collected in an old asbestos-mine; **18—24** proceed from the old magnesite-mine of Cerro Loma Guerra; **23**, **24**, **57**, **243**, **246** and **376** have been taken from isolated fragments of rock. I am not certain whether **53** has been really collected at the locality indicated on the map.

The localities are evenly distributed over the whole eastern part of the island; the western part has not furnished samples, possibly because it has been visited only very cursorily.

SIEVERS (10), AGUERREVERE (1) and HESS (3) mention the occurrence of serpentine on the island. LIDDLE (4) mentions the magnesites, which he regards as contact-products from the periphery of a granite-massif. This is certainly mistaken: the magnesites are found — as elsewhere in the world — in relation with peridotites or serpentines. There are no microscopical descriptions of this group of rocks in the literature.

Five samples are peridotites-in-serpentinization or serpentines, which clearly originated from peridotites. In (**18**) there is still much olivine, each grain with its net-work of serpentine. It is interesting that the original olivines, whose circumferences are recognizable by simultaneous extinction, *have very elongated forms*, which proves that the rock must have crystallized under the influence of thrust-powers. In **191** a considerable content of olivine remains; in this rock, the crystals were isodiametric. At the side of serpentine with net-structure there occur large plates of antigorite which show no trace of orientation. The forming of magnesite has already begun. Samples **225**, **227** and **230** are from the same locality and intimately related inter se. They are serpentines which, by their mash-structure, by the presence of chromite and of bastite (**230**), prove that they originated from peridotites. They contain abundant iron-ore; in 225 veins of opal are present.

As indicated above, the collection contains a large number of serpentines and serpentine-schists. Some of these will be mentioned separately. The rocks **16**, **20**, **21**, **33**, **34**, **48** are coarse antigorite-schists with accessory crystals of magnetite, which are sometimes limonitized. Nos. **26**, **28**, **58** are antigorite-chlorite-schists; **26** contains moreover talc; **58** shows a vein with chalcedone, quartz and opal. In these rocks occur crystalplates, consisting partly of antigorite, partly of chlorite: thus, these two minerals have the

faculty of growing in parallel superposition. No. **190** is an antigorite-jadeite-schist; the jadeite is partly so coarse that pyroxene-cleavage may be recognized, partly it is finely-fibrous, and recognizable only by the great angle of exinction. The antigorite-chlorite schist **186** contains equally some finely-fibrous jadeite. The samples **229**, **232**, **246**, **249**, **250** are ordinary serpentines and serpentine-schists; nos. **229** and **232** very probably originated from a peridotite. No. **60** is a serpentine with advanced formation of limonite and quartz.

Four somewhat abnormal rocks have been found in the northernmost part of the island. No. **17** is a poorly schistose, emerald-green rock. It consists for the greatest part of monoclinic pyroxene which may pass marginally into uralite; there is a matrix of serpentine and magnetite: it is a pyroxene-serpentine-rock. No. **22** is a serpentine-pyroxene-schist with "stream-lined" serpentine and less pyroxene. Still poorer in pyroxene is the serpentine-pyroxene-schist **23** which contains accesorily magnetite; the sections present some veins with opal. Quite another rock is **24** which consists almost entirely of large crystals of bluegreen hornblende with magnetite as an accessory. It has been found as an isolated rock-fragment and may originate from an intrusive dike.

The magnesites **19**, **47**, **53**, **242**, **376**, and the quartz-chalcedone-rock **57** belong to the weathering-products of the serpentines. The magnesites are dirty-white or light-green rocks, often somewhat concretionary. In one of the samples Dr. W. van Tongeren determined the presence of 47 % magnesite. No. **47** presents still, at the side of magnesite, some antigorite. The sample **57** consists of quartz and chalcedone, with remains of serpentine-schist.

The asbestos-containing rocks of Margarita are certainly intimately related with the serpentines. At Manzanillo a beautiful tremolite-asbestos (**14**), an asbestos-schist with antigorite (**15**) and an antigorite-schist (**16**) have been sampled; at the locality **25—28** a chlorite-containing tremolite-asbestos has been found together with antigorite-chlorite-schists. The serpentine-schist **34** and the tremolite-asbestos **35** proceed equally from the same locality. In the field-notes of Mr. Hummelinck asbestos-schists have been mentioned from a point S. from Porlamar; there is no sample from this locality.

At the Morro Moreno three gabbroid rocks (**226**, **228**, **231**) have been sampled in intimate relation with serpentines. The rocks, which have suffered from strong pressure, are saussurite-gabbros.

F. Acid intrusive rocks. Samples: **9**, **93**, **94**, **95**, **96**. These rocks are very rare, and their character is not absolutely certain. Liddle (4), in his section, and Aguerrevere (1) mention granitic rocks from Paraguachi, without giving details.

No. **9** is a white, non-schistose rock, containing large feldspars. The slide presents: 1. various large microclines with "quartz-drops", 2. some large orthoclase-perthites, 3. many large albites, frequently filled with

sericite. These three minerals may show traces of idiomorphism. At the side of them there is a fabric of quartz, acid plagioclase and microcline in smaller crystals; the intergrowth may be aplitic. The quartzes are strongly undulatory, at many points even strongly cataclastic and crushed along shear-planes. The rock seems to be a dynamomorphic aplite. It was found near Paraguachi, from where LIDDLE and AGUERREVERE mentioned their "granites".

Samples **93—96**. The rocks are poorly schistous. They show in the slides: 1. porphyritic albites with traces of idiomorphism, often strongly bent, broken and crushed at the margins, 2. quartz, partly in large crystals with undulatory extinction and with marginal crushing zones, partly also in totally crushed zones along shear planes, 3. sericite, epidote, zoisite and chlorite concentrated in small green streaks. The rocks are strongly dynamometamorphic albite-aplites.

G. Lamprophyric rocks. Sample 12 is possibly a strongly altered biotite – amphibole – lamprophyre.

Basement of Coche. AGUERREVERE has mentioned from the NE. part of Coche "una roca cristalina verdosa, probablemente metamorfica" (1, p. 400). HUMMELINCK has visited only the SE. part of the island, the area of Guamache, where he sampled three sericite-quartzites of the basement (**166**, **167**, **168**), covered by a young conglomerate.

General considerations on the Basement-Rocks of Margarita and Coche. Most of the rocks of the basement of Maragarita are para-schists. As such may be regarded all the mica-schists, most, if not all of the gneisses, and all the crystalline limestones and calcite-bearing schists. Intrusive rocks and ortho-schists are, of course, the aplites, the problematic lamprophyre, the gabbros, the peridotites and those serpentines which clearly have originated from peridotites. For the other serpentines and related rocks, and for the "various basic schists" it is impossible to say, whether they are ortho- or para-schists, although it is probable that many of them are ortho-rocks. Thus, we can distinguish in the basement of Margarita a series of older rocks, the para-schists, and a series of (somewhat) younger rocks, the (intrusive) ortho-rocks. The series of para-schists seems to be quite homogeneous and not to comprise rocks of different cycles: the different kinds of rock are clearly linked by transitional types. Gneiss, micaschist, calcite-bearing schists and quartzite (not sampled) are alternating at locality **1—7**; marbles, different schists and quartzites (not sampled) are intimately connected in the region S. of La Asunción. But also the ultrabasic schists, part of which must have ortho-character, seem to be related intimately with the foregoing. This holds good for the association gneiss-serpentines at P. Manzanillo and in the area S and SW from La Asunción. The only rock which does not fit well into the metamorphic complex is the graphite-

bearing quartzite no. **97** (?ex chert), S. of Porlamar, which might belong to a younger cycle.

Most of the rocks of the basement of Margarita are also known from the Caribbean Coast-Range of Venezuela. Gneisses and micaschists, comparable with those of Margarita, have been described from the roads La Guaira-Caracas and Puerto Cabello-Valencia (6). Mr. HUMMELINCK has collected samples at Esmeralda and Puerto Santo near Carupano, of which I mention: a micaschist with chlorite, a gneissic micaschist, an albite-gneiss and dynamometamorphic albitites. Rocks comparable with the marbles and the calcite-schists have been found along the road La Guaira-Caracas (6) and at Puerto Santo: coarse marbles without accessories, schistose marbles with graphite, quartz, albite, epidote and chlorite, and basic schists with calcite-layers. Serpentine-schists are equally known from the road La Guaira-Caracas, and from the mainland, S. of Margarita, where, at Manglillo, samples of a serpentine-talc-rock, of non-schistose, somewhat opalized antigorite-serpentine, of quartz-chalcedone-rock and of altered gabbro have been collected by Mr. HUMMELINCK.

I am not certain whether the few rocks from Coche, all sericite-quartzites, belong to the same cycle as the basement rocks of Margarita, where this type is practically absent. Only the southernmost sample from Margarita (97) is an (aberrant) graphite-quartzite. On the very small islands S. of Coche (Lobos, Isla de Caribes) and at Chacopata the same rocks as on Coche have been collected: sericite- and muscovite-quartzites (**170**, **171**, **173**, **174**, **181**, **183**, **184**, **185**), whilst rocks of the "Margarita-type" (serpentines and micaschists) are found somewhat more to the S. at Manglillo. It is evident that the sericite-quartzites occupy a well-defined area, where other basementrocks seem to be lacking.

I am inclined to regard the quartzitic rocks of Coche and of Chacopata-Lobos as metamorphic Cretaceous and the other rocks of the metamorphic basement as older. The quartzites are not materially different from lower cretaceous sandstones in different parts of Venezuela (RUTTEN, 8, p. 345); the basement-rocks of Margarita, with their many basic intercalations (and/or intrusions) and their lack of what might be called "indicative cretaceous stratigraphy" (i.e. the existence of a sequence: sandstones-limestones-shales) would be older. It is certain that the basement rocks are all pre-eocene, as the non-metamorphic Eocene of Margarita contains pebbles of the basement rocks. If the age, contributed to gabbroid rocks of Lara by RUTTEN (7) and to serpentines of the Serranía del Interiór by SCHÜRMANN (9) is right, basic and ultrabasic rocks of different age must exist in North Venezuela and Margarita.

In E. Margarita Mr. HUMMELINCK has measured a considerable number of strikes and dips. A glance at the map shows that by far the most strikes are NE, the dips SE. From the field-notes it can be concluded that plication is frequently visible in the outcrops (f.i. Manzanillo, P. Ausente, Tetas de Maria Guevar, Morro de Robledor). Thus we may not regard

the SE-dipping beds as the SE-wing of a gigantic fold, *but certainly we have to do with isoclinal folds*, the details of which can, however, not been unravelled with the available data. It must be observed that the NE-strike which is the oldest visible tectonic direction in Margarita and Coche tends to cross the much younger direction which is indicated by the row of islands Aruba-Los Testigos (7). *It is, therefore, quite probable that the metamorphic basement continues unto unknown distances below the Caribbean Sea.*

In relation with the tectonic processes two facts must once more be mentioned: first that one of the peridotites has crystallized under the influence of thrusting forces, second that *almost all the rocks of the basement show signs of crushing metamorphosis, posterior to the crystallization metamorphosis and, of course, prior to the deposition of the Eocene.*

Eocene of Margarita. Sediments in the region of Pampatar and Porlamar, already known to WALL (11), have been called Cretaceous by LIDDLE (4), Midway by MAURY (5) and Eocene by AGUERREVERE (1). Mr. HUMMELINCK has collected samples in Pampatar, Gaiquire and Punta Mosquito. The rocks are sandstones, greywacke-sandstones, shales, conglomerates and some limestone; the age is eocene. There is, however, a rather marked difference between the rocks of the three localities, and they will therefore be described separately.

The rocks of Pampatar and Punta Ballena (**61—68**) are grey or green, brown-weathering, platy, calcite-bearing sandstones and shales. Almost all of them contain Globigerina's and ?Radiolaria; no. **62** contains a fragment of a Lepidocyclina. The clastic minerals are: quartz, plagioclase, muscovite, biotite, serpentine and chloritic minerals. There are in the sandstones fragments of phyllite, quartzite, quartz-muscovite-schist, intergrowths of quartz and albite and cherts. The samples **203—208** proceed from the region W. of Pampatar. The calcite-bearing sandstones (**203, 206, 207**) contain very rare, smaller Foraminifera, quartz, plagioclase, muscovite, serpentine; **207** contains grains of schists and of porphyritic material. **204** is a marl, **208** a limestone without fossils. No. **205** is a conglomerate with a very interesting association of pebbles: black chert, many porphyritic fragments, plagiaplite and quartzepidote-rocks which are very similar to rocks, associated in Aruba with the quartzdiorites.

On Gaiquire, which is a small island within a lagoon N. of Morro Moreno Mr. HUMMELINCK collected the sandstones **211, 212, 213, 219** (with clastic: quartz, plagioclase, muscovite, serpentine, quartzite, chert, porphyrites and fine-grained schist) and the conglomerates **216** and **223** (with pebbles of porphyrites, cherts, quartzite and sericite-quartzite). The numbers **214, 217, 218, 220—222** and **224** are pebbles from the conglomerate; they are porphyrites, amygdaloidal porphyrites and cherts. Sample **215** is a dark limestone. Sample **210** is a conglomerate with Orbitoids, probably belonging to the genus Discocyclina.

The samples **99—108** proceed from Punta Mosquito. According to the fieldnotes of Mr. HUMMELINCK there occur here conglomerates with rapidly changing thickness, sandstones and limestone. At the coast fine, small anticlines are to be seen. The pebbles in the conglomerates are quartzite, schisty quartzite, black chert, true radiolarite, quartzdiorite and porphyrite. The fossils are Corallinaceae, Globigerinae, small Camerinidae, Lepidocyclinae and Discocyclinae. It was possible to isolate a lot of Orbitoids from a sample and to recognize the presence of Discocyclina georgiana Cushman and of Lepidocyclina trinitatis H. Douv., proving the eocene age of the deposit.

To the Eocene belong probably also the samples **109** and **110** from Isla Blanca, S. of Pampatar, being a phosphatized marl and a phosphatized sandy limestone with Globigerina.

The rocks of the Eocene are in the first place of interest by the clastic material which they contain. Part of it has been clearly derived from Margarita; many components must, however, proceed from elsewhere. The cherts and radiolarites which very probably are of cretaceous age, the porphyrites and the quartzdioritic rocks can not come from Margarita. The porphyrites and quartzdiorites may come from the North: we find similar rocks on Los Frailes and Los Testigos; the origin of the cherts is unknown.

The eocene rocks are rather strongly folded; in the area of Pampatar they seem to form a syncline; it is quite probable that they have been separated from the basement of Margarita — as indicated in LIDDLE's section (4) — by a fault.

The youngest "capping" deposits. It seems that the whole of Cubagua and a large part of Coche is covered by young marine, partly detritical deposits [1]). Their elevation above the sea to about 200 ft must have occurred in subrecent times. An elevated plateau with detritical, marine deposits covers a large part of Western Margarita; in the midst of it arises the mountaineous massive of Macanao (American Admiralty Chart 2035). According to the field-notes of HUMMELINCK 15—25 m of detritical deposits are to be seen to the West of Boca del Rio. He is of the opinion that, farther in the interior, this plateau rises to about 200 m; it is dissected by deep canyons. Slightly elevated marine deposits are found S. of the Laguna Arestinga, at the coast near San Juan Griego, at Punta Ausente and N. of Punta Mosquito. It is clear from the foregoing that different parts of Margarita have been elevated in subrecent time to different heights.

The petroleum seepages in Margarita and Cubagua. A rather impor-

[1]) A small collection of quaternary molluscs of Cubagua will be studied by Miss T. VAN BENTHEM JUTTING in Amsterdam; Prof. GERTH, of Amsterdam was so kind as to determine the following corals from the island: Millepora alcicornis (L), Orbicella acropora (L), Oculina diffusa (L), Siderastraea radians Pall.

tant seepage of oil must exist to the NW. of Laguna Chica in Western Margarita; Mr. HUMMELINCK got a bottle of the heavy brown oil, but did not visit the locality. From NW. Cubagua he collected some samples of sand, strongly impregnated with oil. We may presume that in the underground tertiary or cretaceous strata are to be found and that there exist important faults along which the oil has migrated to the surface. The presence of tertiary strata in the underground would be in accordance with the occurrence of Eocene on Margarita; the occurrence of cretaceous strata would not be strange, given the occurrence of clastic cretaceous material in the Magdalena Eocene. *The situation of the oil-bearing strata in the strike of the large oilbearing geosyncline of Lara-Falcon is quite natural.*

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Mr. HUMMELINCK's collections are in the Geological Institute of the University of Utrecht; details are to be found in the year-catalogue for 1940.

Palæontology. — *The fossil human remains discovered in Java by* Dr. G. H. R. VON KOENIGSWALD *and attributed by him to* Pithecanthropus erectus, *in reality remains of* Homo sapiens soloensis 1) *Continuation.* By Prof. EUG. DUBOIS.

(Communicated at the meeting of June 29, 1940.)

In order to give a still somewhat better idea of the dentition of Homo wadjakensis II, I reproduce in Plates III and IV, natural size, the telephotographic outlines of the fossil maxilla and mandibula, published ½ natural size in 1920 (l.c.), before all the fossil pieces found were united, especially the right ramus with the corpus mandibulae, and 6 teeth, dropped on the spot, were inserted. Clearly these accurate outlines of the original fossils, including the occlusal views, (together with the two views of the casts of the completed upper and lower jaw, reproduced in Plate II) show the important fact that there did n o t exist any diastema, and that the upper canine did n o t penetrate between the lower canine and first premolar. In my eyes VON KOENIGSWALD's and WEIDENREICH's photographic reproductions of the upper jaw of "Pithecanthropus" skull IV, of Sangiran, January 1939 2), reproduced in natural size in the annexed Plate V, shows the same thing. The small seeming diastema of the authors can be put to account of the damaged and incomplete alveoli, the said penetration of the canine to account of its sagging.

It is interesting to compare with the dentition of VON KOENIGSWALD's finds, and the Solo-man skulls, a similar but elaborately described find of another fossil man related to the present Australian race: the skull of Talgai in Queensland, Australia, which, discovered in 1884, was elaborately described in 1918 3). This skull of a "male youth" (for m^3 was still

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1) Corresponding to this alteration of the title, the following corrections in the text of my paper of March 30, 1940 (Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam **43**, 494—496 (1940)) may be inserted here:

P. 494, line 13 from below, instead of **is identical** read **belongs to the same group**
„ 495, „ 10 „ above, cancel **Homo wadjakensis II and**
„ 495, „ 25 „ „ read **an exact Solo-man skull.**
„ 495, „ 3 „ below, instead of **not** read **beit much**
„ 495, „ 2 „ „ place a note 1) behind **II**: Concerning Sinanthropus pekinensis, see FRANS WEIDENREICH, The mandibles of Sinanthropus pekinensis: a comparative study. Palæontologia Sinica. Ser. D, Vol. VII, Fasc. 3, p. 33.

2) G. H. R. VON KOENIGSWALD und FRANZ WEIDENREICH, The relationship between Pithecanthropus and Sinanthropus. "Nature", vol. 144, pp. 926—929. Dec. 2, 1939.

3) STEWART ARTHUR SMITH, The fossil human Skull found at Talgai. Queensland. Philosophical Transactions of the Royal Society of London. Series B, Vol. 208, pp. 351—387. [Plates 12—18]. 1918. — See also EUG. DUBOIS, The proto-australian fossil man of Wadjak, Java. Proc. Kon. Akad. v. Wetensch., Amsterdam **23**, 1013—1051 (1920), p. 1028.

PLATE III. Homo wadjakensis II.

Nat. size.

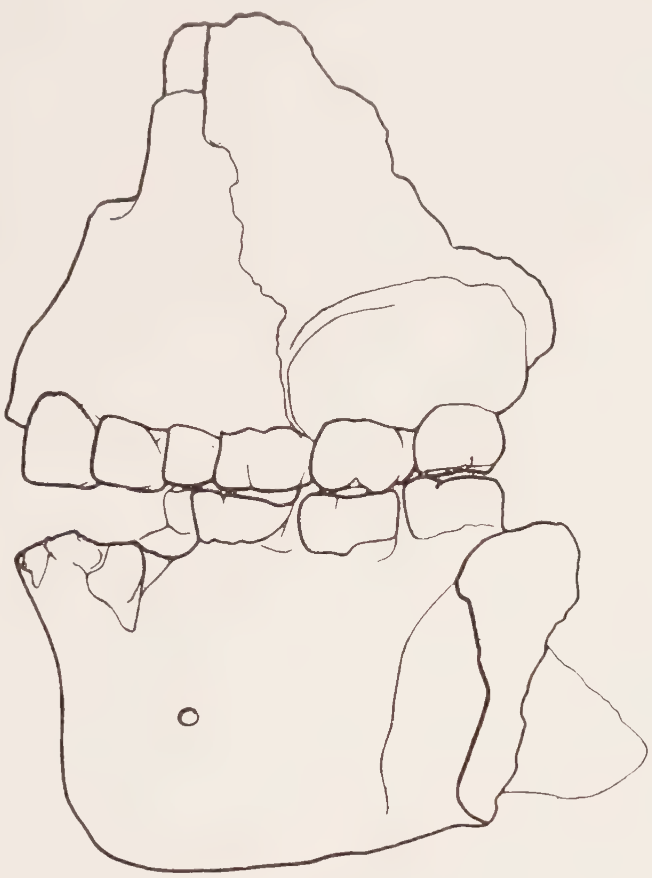

Fig. 1.
Left-side view of Upper and Lower Jaw.

Fig. 2.
Front view of Upper and Lower Jaw.

PLATE IV. Homo wadjakensis II.

Nat. size.

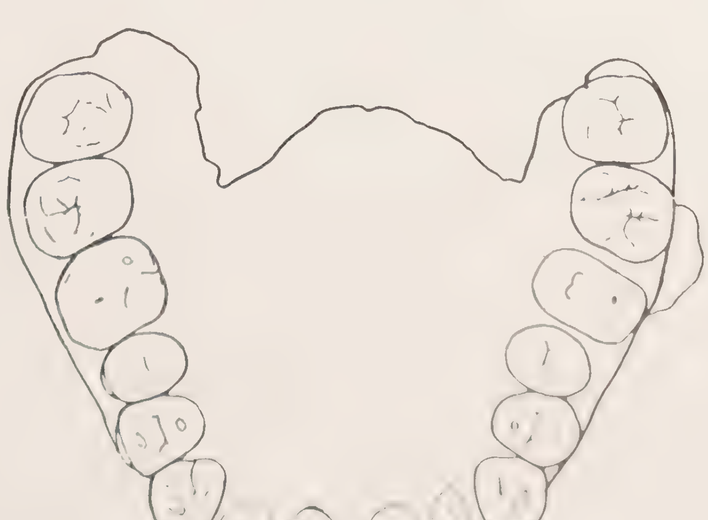

Fig. 3
Palatal view of Upper Jaw

Fig. 4
Lower Jaw, seen from above

unerupted), though cracked *in situ* into numerous fragments, more or less considerably dislocated, but held in position by thin layers of calcareous earthy matrix cementing them together, the condition resembling a coarse mosaic, can yet be clearly recognized as not deviating in its general features from the present aboriginal Australian skull. The cranium as a whole, and the palatum, however, hardly admit of any reliable measurements. These can still be made at the tooth-crowns, each in itself, but most of them have more or less receded from each other; the a p p a r e n t palatal area thus considerably exceeds the r e a l, which, in my opinion, was no larger than that of the Australian native of present times. SMITH supposes that the upper canine, in an analogous way as in the dentition of the Apes, though without a true diastema in the maxilla, penetrated, almost ape-like, with its apex between the lower canine and the lower first premolar.

In my opinion there is reason to doubt this, on the ground of a comparison with the teeth of Wadjak II, especially on account of identified contact facets on the Talgai and the Wadjak upper canine. In his reconstruction, SMITH lowers the upper canine till the upper border of its crown comes very nearly on a level with the upper border of the crown of the premolar. Erroneously, for the crown-border of such a large upper canine as the Talgai one is always considerably above the level of the crown-border of the first upper premolar; in the maxilla of Wadjak the distance is 3 mm. This upper canine, therefore, cannot, in life, have projected so far downward. The canine of Wadjak II, which strikingly resembles that of Talgai, is also of equal breadth as the latter, and if its wear were as little advanced as that of the canine of the boy of Talgai, it would no doubt be as pointed and little shorter than the latter. For these reasons I cannot agree with SMITH in ascribing to the fossil skull of Queensland, which he too considers as typically Australian, "a human dentition in which these anthropoid characters are manifested in a manner quite unknown in man, except for the single example of *Eoanthropus*".

Now, twenty-two years after STEWART ARTHUR SMITH's publication, we observe, in a strikingly similar case, VON KOENIGSWALD and WEIDENREICH led astray to the same mistake, by the same belief in evolution from Ape to Man through g r a d u a l t r a n s f o r m a t i o n o f p a r t s o f t h e b o d y. This belief, in matters of evolution, results from almost exclusive m o r p h o l o g i c a l consideration of the organisms. Hence the apparent inconsiderate supposition of those investigators, that the large, bestial, canines of the Apes are mainly organs of defence, and became gradually smaller and less apelike in the course of Man's phylogenetic development. Apparently, however, according to studies of the animals in nature, the large canines of the Apes are not principally organs of defence, in contrast to the canines of Man, but are specially adapted to the kind of food of those animals. Indeed, many years ago, EMIL SELENKA expressed this opinion with regard to the Orang Utan, studied in Borneo. If that is

true of the Anthropoids in general, as probably also of the Monkeys, and even, after all, of the Carnivores, such a gradual transformation from Ape to Man is hardly conceivable.

In the same communication on the relationship between *Pithecanthropus* and *Sinanthropus* ("Nature", l.c., pp. 928—929), VON KOENIGSWALD and WEIDENREICH give clear expression to their implicit belief in human evolution by gradual transformation, in the lines I may quote here:

"Considered from the general point of view of human evolution, Pithecanthropus and Sinanthropus, the two representatives of the Prehominid stage, are related to each other in the same way as two different races of present mankind, which may also display certain variations in the degree of their advancement.

"The Prehominids are separated from the Neanderthal group by a considerable gap. On the other hand, an apparently close relationship exists between Pithecanthropus and *Homo soloensis*, the skulls of the latter appearing like an enlarged form of the former. Certain peculiarities of Pithecanthropus reappear in exactly the same form in *Homo soloensis*. Those traits which suggest an already more advanced type, like the greater cranial capacity, and several other structural features, can be derived directly from Pithecanthropus, and correspond to the condition in the Neanderthal stage already attained by *Homo soloensis*. The two available fragments of the tibia of *Homo soloensis* show no special particularities, with the exception of a pronounced platymeria, exhibiting only recent human characters in their general form and in details.

"The finds reported herein show that Java has become the most important centre for the study of Prehominid forms. Not only Prehominids, but also the following evolutionary stage, *Homo soloensis*, are represented there. Furthermore, we know that the Wadjak man of Java represents another early form of recent man, whose upper jaw (Wadjak II) displays in some respects a most surprising resemblance to the Pithecanthropus upper jaw."

In reading this, we may not lose sight of the insufficiency, in geological as well as in morphological respect, of what we learned from the descriptions about the true nature of the fossils, as they were found by the collectors of VON KOENIGSWALD, or himself. This insufficiency, indeed, concerns the exact geological localization of the finds, as well as the original condition of the fossils before the restoration. Apparently, VON KOENIGSWALD, in his opinion about the true nature of his finds, does rely more on (insufficient) stratigraphical data than on unprejudiced examination of the fossil remains themselves. No wonder that his implicit belief in human evolution from Ape to Man, by gradual transformation of parts of the body, according to the Darwinian point of view, would lead him astray about the true nature of his finds, and wrongly guide the hand which made the restorations.

Indeed, on closer examination, the fossil human remains attributed by VON KOENIGSWALD to Pithecanthropus, appear all of them to be, owing to

their morphological character, without any doubt remains of *Homo soloensis*.

This result of morphological examination does not fit in with VON KOENIGSWALD's opinion that his fossils belong to the Trinil layers, or even (the Modjokerto child skull) an older layer than these.

But circumstances of regional geology, as well as facts concerning the character and state of fossilization and the external appearance of the fossils, properties which they have in common with the Talgai skull, evidence that the skulls and jaws of VON KOENIGSWALD's "Pithecanthropus" belong to a relatively much later geological age than do the layers in which (in the years 1890, 1891, 1892, 1897 and 1899) I found real *Pithecanthropus erectus* fossils. In the regions where the remains of *Homo soloensis* were found, at Ngandong, and of VON KOENIGSWALD's "Pithecanthropus", at Sangiran, limestones of older formations come to the surface, to absolute higher levels than the Trinil layers. These limestones actually contributed to the "matrix" (composed for the greater part of detritus from Trinil rock) of those human fossils, and determined character and state of the fossilization, which, as observed on the fossils of Sangiran and Talgai, were unable to resist the influences of wear in such an effective manner as did Trinil fossils.

Thus, also in geological respect VON KOENIGSWALD's finds are different from the real Pithecanthropus fossils. They are, doubtless, remains of *Homo soloensis*.

Concerning *Homo wadjakensis* and *Sinanthropus pekinensis*, we may remark that, although the character of fossilization is not different from that of *Homo soloensis*, the conditions of preservation at Wadjak and Choukoutien were better.

Naturally, I perfectly agree with VON KOENIGSWALD and WEIDENREICH in regarding *Homo wadjakensis* and *Homo soloensis* both as early forms of recent man (*Homo sapiens*). These forms, however, differ in some important morphological characters. Indeed, the Wadjak skulls lack the platycephaly, the torus supraorbitalis, the strongly expressed occipital slant, those conspicuous features of every Solo man, as also of "Pithecanthropus" Skull IV. On the other hand, Wadjak man Skull II exhibits a very pronounced chin, whereas, apparently, *Homo soloensis* possessed only a rudimental one, comparable with the trigonum mentale, which WEIDENREICH (l.c.) found in lower jaws of *Sinanthropus pekinensis*. The upper jaw, however, was probably of similar form in Solo man and Wadjak man; therefore that part of the skull of Wadjak man II displays "a most surprising resemblance" to the upper jaw of "Pithecanthropus" Skull IV, which really is a *Homo soloensis* skull.

Nevertheless, we may regard the two early forms of *Homo sapiens*, proto-australians of Java, as nearly related (they may even, as it seems, geologically belong to quite the same age), because those distinctive morpholo-

gical characters, as THOMAS HENRY HUXLEY [1]) said in the case of the Neanderthal cranium, "did not extend deep into the organization" — thereby, undoubtedly, meaning that they did not directly concern the animal organization.

Therefore, the morphological characters, especially of the supraorbital region and the occiput of the Wadjak II Skull, in contrast to the Wadjak I Skull, approach somewhat Ngandong skulls (*Homo soloensis* I and V), though the first Wadjak skull had come to light near the spot where in the next year the second skull was dug out, so that they are certainly geologically contemporaneous.

In this second Wadjak skull, the very prominent and low-lying arcus superciliares meet with a likewise swollen glabella, forming with it a much broader protuberance than the proper glabellar one, and melt together with the medial part of the orbital arch, as far as about the incisura supraorbitalis. The shape of the orbit is thereby made rectangular. Accordingly, there is above that broad protuberance, in the same breadth of ample 60 mm, a real sulcus supraglabellaris, the beginning of a sulcus supratoralis. There is, however, no fossa supraglabellaris, which is so characteristic a feature of the Neanderthalian supraorbital region.

The same form of the supraorbital region of Wadjak Skull II, now, is of frequent occurrence in Australian-aborigine skulls. Some of them approach in this respect the Neanderthal type still more, as far as to develop a real torus supraorbitalis reaching to the processus zygomaticus.

Of the Ngandong skulls (*Homo soloensis*) I and V certainly do this. Ngandong Skull VI, on the contrary, exhibits a transversally depressed glabella, thus entirely lacking the proper glabellar protuberance of Wadjak Skull II, and also of Wadjak Skull I. Nevertheless, Ngandong Skull VI possesses a moderately pronounced but real torus supraorbitalis, although this is interrupted in the middle. In many present-Australian skulls, also, the glabella is somewhat depressed transversally.

Apparently the different conditions of the supraorbital region in skulls of Australian aboriginals [2]) are represented, more or less separately

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1) Evidence as to Man's Place in Nature. London 1863. P. 157: "So large a mass of brain as this, would alone suggest that the pithecoid tendencies, indicated by this skull, did not extend deep into the organization."

2) HERMANN KLAATSCH, The Skull of the Australian Aboriginal. Reports from the Pathological Laboratory of the Lunacy Department. New South Wales Government. Vol. I. Part. III. Sydney 1908.

With this paper KLAATSCH, initiating investigations "based on principles of evolution", intended dealing with the skulls of Australian aboriginals, with special reference to specimens obtained by Dr. W. E. ROTH from different parts of Queensland. These skulls, comprising those of about ninety individuals, and deposited in the Australian Museum, Sydney, are of very great value in that their full-blood origin is assured. With a view to giving a general outline of the variations met with throughout a large number of the series, KLAATSCH has found it necessary to restrict special descriptions and

(selectively), in the Wadjak and Ngandong skulls, most strongly in the latter. The same we observe concerning the development of (1) the slanting nuchal plane; (2) the deviation of the dental arch from the horseshoe form and resemblance to the form of the upper dental arch of Homo wadjakensis II (Plate V, Fig. 3), this being only relatively much broader — length-index **74** (identical with that of Rhodesian man), in contradistinction to **90** of KLAATSCH's Australian R. 69 —; (3) the very pronounced chin. The existence of a trigonal mental prominence is the rule in the mandibles of Australian aboriginals [1]); (4) the platycephaly.

Abundant evidence about this varied skull morphology of Australian aboriginals is available in the collections of the Australian Museum, Sydney, of which KLAATSCH has made an elaborate study [2]).

The development of these features — in the opinion of the majority of anatomists — is a consequence of their physiological significance. According to the excellent investigations of TOLDT [3]), this development is directly correlated with that of the jaws in general, and the different parts of the apparatus concerned in mastication in particular. Accordingly they develop only at the same time with those parts, during the ontogeny. The pressure which, in the act of masticating, is exercised by the teeth on the upper jaw bone, is transferred to the frontal bone by three pillars: a nasal one and two jugal pillars. If the inferior frontal region is situated approximately in the direction of this pressure, it remains unchanged; with receding forehead and flattened skull (platycephaly), however, reinforcements in that frontal cross-beam come into being, especially in the case of a strong dentition.

Concerning the p l a t y c e p h a l y, we observe that, amongst recent races of man, the Australian aboriginal possesses the minimum of calvaria-

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illustrations to certain specimens which showed differences sufficiently great to warrant them being considered typical. —

Compare also the studies of D. J. CUNNINGHAM, The Evolution of the Eyebrow Region of the Forehead, with special reference to the excessive supraorbital development in the Neanderthal race. Transactions Roy. Society of Edinburgh. Vol. 46, Part II, No. 12, 1908. — Furthermore, the earlier studies of G. SCHWALBE: Studien über Pithecanthropus erectus. Zeitschrift für Morphologie und Anthropologie, Band I, 1899; also: Der Neanderthalschädel. Bonner Jahrbücher, Heft 106, 1901, and diverse other papers.

[1]) Concerning the Trigonum mentale, see: HANS VIRCHOW, Die menschlichen Skeletreste aus dem Kämpfe'schen Bruch im Travertin von Ehringsdorf bei Weimar, Jena 1920, pp. 51—64: Kinngegend. — Concerning the Australian aboriginal's dentition I may quote: T. D. CAMPBELL, Dentition and Palate of the Australian Aboriginal. Thesis, University of Adelaide, 1925. See particularly illustrations of **squarish front portion** of the dental arch, and of **parallel arch form**; the first feature, with a deviation from the horse-shoe form, is met with in the upper dental arch of Wadjak II; the parallel arch form is the extreme development of that deviation.

[2]) See note 2 of foregoing page.

[3]) C. TOLDT, Brauenwülste, Tori supraorbitales, und Brauenbögen, Arcus supraciliares, und ihre mechanische Bedeutung. Wien 1914. Mitteilungen der Anthropologischen Gesellschaft in Wien. Band 44.

height-index in relation to the glabella-inion line, (SCHWALBE's Kalottenhöhe-Index), 45, according to the reports of BERRY, ROBERTSON and STUART CROSS [1]), for 100 Australians, which minimum thus is below the index, 52, of Wadjak Skull I, whereas, according to the same reports, the medium in the Australian aboriginal race is 53 and the maximum 62. Wadjak Skull II, judging from the skull fragments, was possibly somewhat more flattened than Wadjak Skull I. Ngandong Skull VI, the only one of Homo soloensis which is so well preserved as to allow accurate measurements, in this respect, has an index 42; the index of the less well preserved Ngandong skulls I and V could not, apparently, differ much from this. Thus, it appears that the flattening of the skull in *Homo sapiens soloensis* was equal to that in *Homo neanderthalensis* — with calvaria-height-indices from 40 to 44.

Similar observations obtain with regard to a feature in the occipital region of the human skull, of frequent occurrence but different development: the torus occipitalis — being a swelling of the field between the lineae nuchae superiores (lineae nuchales terminales) and the lineae nuchae supremae (lineae nuchales supraterminales) — in recent races, especially again, in the Australian aboriginal, and the fossil races in consideration, comparatively the fossil species *Homo neanderthalensis*. It is, moreover, noteworthy that in Australian aboriginal skulls, alike in Ngandong Skull VI, only the medial part of each torus occipitalis half is developed.

Obviously, all those differences of features and conditions of homologic parts of the skull, in the Australian aboriginal, in Wadjak man and Ngandong man, comparatively in *Homo neanderthalensis*, have only mechanical (direct physiological) significance; they do not correspond to different stages of human evolution.

As Peking man (*Sinanthropus pekinensis*) has so many peculiarities in common with Ngandong man (*Homo* or *Javanthropus soloensis*) [2]) that both must be considered unquestionable members of the same proto-australian group, differences of features and conditions of those homologic parts of the skull neither correspond here to a different stage of human evolution. These morphological differences refer principally to a still more pronounced development of the torus supraorbitalis than in *Homo neanderthalensis*, and a correspondingly excessive flattening of the skull cap — the calvaria-height-index, in relation to the glabella-inion line, being so

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[1]) See quotations in: EUG. DUBOIS, The proto-australian fossil man of Wadjak, Java. Proc. Kon. Akad. v. Wetensch., Amsterdam **23**, 1018 (1920).

[2]) To the arguments given in my papers, quoted on p. 496 of the first part of this communication, may be added the demonstration of the similarity of the endocranial casts by C. U. ARIËNS KAPPERS in "The endocranial casts of the Ehringsdorf and *Homo soloensis* skulls". Journal of Anatomy. Vol. **71**, Part I, October 1936. Cambridge University Press. pp. 67—75.

low as 36 to 41, according to my measurements on the published representations of the skulls.

Another proof of Peking man belonging to one and the same group of proto-australians with Ngandong man and Wadjak man, is given again by the upper and the lower jaw. Up to the present there is only one upper jaw of Sinanthropus pekinensis available, but this, though slightly damaged, shows the features in question with sufficient distinctness. It is described and figured in WEIDENREICH's elaborate treatise on the dentition of Sinanthropus pekinensis (1937) [1]), Text, p. 136; Atlas, Fig. 345. Here, unmistakably again, appears a striking resemblance to the upper jaw of Wadjak man II. For comparison of the dentition in the lower jaw, there is hardly anything more available of Sinanthropus than the alveolar arch of one specimen of sufficient lower jaw. But this shows at least probable similarity in the very large breadth-index of the dentition. [2]) It is unnecessary to repeat the occurrence of the trigonum mentale in Sinanthropus, characterizing *Homo sapiens*.

Concerning the bearing on human evolution of the supposed similarity of the skulls of *Pithecanthropus* of Trinil and the (first) *Sinanthropus* of Choukoutien, implying near relationship, nay generic identity, it may now be observed that the real morphologic evidence is completely inconsistent with this supposition, which, indeed, is only based on the overlooking of this evidence.

Whereas, from the first, the gibbon-like features of the *Pithecanthropus* skull has always drawn the attention of those unbiassed by the preconceived opinion of human evolution by gradual transformation of parts of the body, others — and they were not a few — would not see those features, even their suspicion of prejudice fell on them who could not deny the existence of the features in question.

In reality the morphologic similarity of the large skull cap, excavated from a volcanic tuff, at Trinil, in 1891, and described in 1894 as *Pithecanthropus erectus*, to the homologous part of the skull of a small *Hylobates* species is most striking. Not only the median contour, but also the special form of the torus supraorbitalis and of the lower tabular part of the occipital bone, such as it was before it was naturally damaged in the fossil state, is quite gibbon-like. An absolutely distinctive gibbon-like feature, however, is preserved, just in that damaged region of the planum nuchale. One feature in this region the high situation of the linea nuchae inferior (linea plani nuchalis), in the Gibbons and the other Anthropomorpha, is very different from that in Man. Another feature, on the contrary, distinguishes the gibbons — *Hylobates* and *Symphalangus* — from the large anthropomorpha; this is the development, in the gibbons, of a deep

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1) FRANZ WEIDENREICH, The dentition of *Sinanthropus pekinensis*. Palæontologia Sinica. New Series. D, No. 1. Peiping 1937. Text and Atlas.

2) FRANZ WEIDENREICH, The mandibles of Sinanthropus pekinensis. Ibid. Series D, Vol. 7, Fasc. 3. Peiping 1936. See especially p. 100, Table XV.

depression below about the inner third of the linea nuchae inferior (linea plani nuchalis), that part of the linea, and the depression below it, serving for the insertion of the musculus rectus capitis dorsalis minor. In this respect Chimpanzee and Orang-utan are different, even more than Man. In *Hylobates* and *Symphalangus* the linea plani nuchalis is very much nearer the linea nuchalis terminalis than in Man; however, in *Symphalangus* it deviates relatively more at the crista occipitalis externa than laterally. This medial deviation is more considerable in Chimpanzee; but this feature there is variable, the deviation may be over a longer extent, as it is regularly in the Orang-utan, here with varying distance from the linea terminalis.

Now, the skull cap of *Pithecanthropus erectus* having lost in this region much of the superficial bone substance, with the crista occipitalis externa and all those lines, it has, nevertheless, preserved a large and deep united depression, corresponding to the place of insertion of the musculi recti capitis dorsales minores. The entire region has, undoubtedly, once been quite gibbon-like, but all the features had dimensions, the double of those of a present Siamang.

Needless to insist on the gibbon-like appearance of *Pithecanthropus erectus*, and on the unfoundedness of the conception of human evolution by gradual transformation of Chimpanzee-like ancestors.

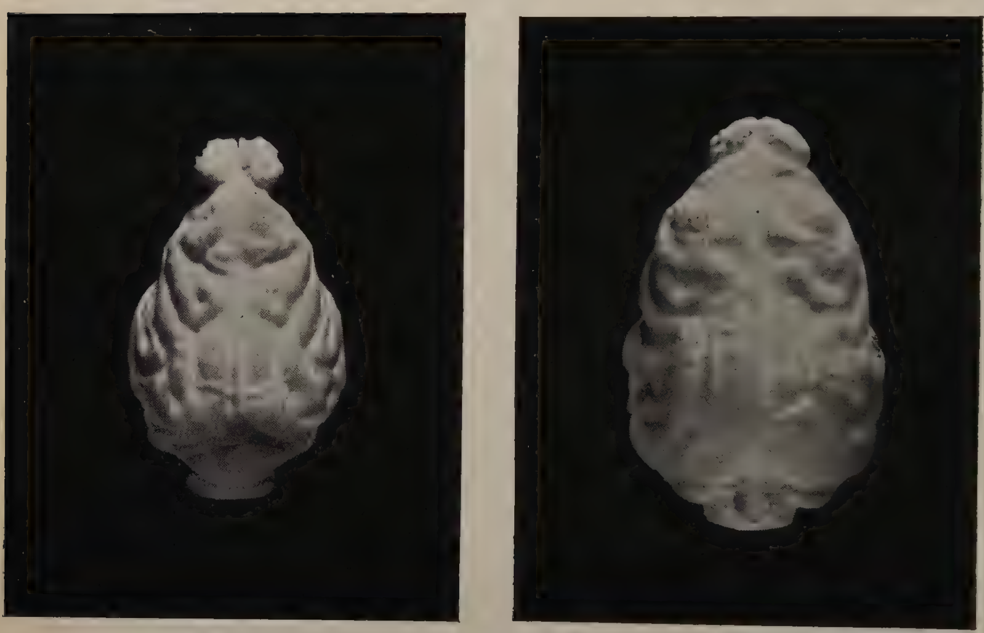
The most important conclusion of a close examination of the Trinil skull cap is that the so-called *Sinanthropus pekinensis*, an indubitable *Homo*, is no near relation of *Pithecanthropus*, to say nothing of identity. The resemblance in the median sagittal contours of the two crania, which have about equal capacity, cannot surprise us, taking into consideration the mechanical significance of platycephaly. There are significant differences, as, above all, concerning the parietal vertex of the brain, a human distinctive which *Pithecanthropus* lacks, as do the Anthropomorpha and Monkeys.

As to the now known limb bones of *Sinanthropus*, they are, also in the eyes of WEIDENREICH, quite human; particularly the femur is in every respect of the *Homo sapiens* type, and the dimensions are such as correspond to the small cranial capacities. The six thigh-bones which we now have of *Pithecanthropus erectus*, all of them showing in the preserved parts the same morphologic deviations from the human femur, are, on the contrary, large in proportion to the cranial capacity. This does not fit in with the opinion that they might be human thigh-bones.

Concerning the teeth attributed to *Pithecanthropus erectus* one may refer, for the first lower premolar, to the resemblance to the homologous tooth of the gibbon-like *Propliopithecus* from the Oligocene of Egypt. As to the upper molars, it may be remarked that the shape of their crown not only resembles that of the homologous molars in the Orang-utan, but also, in some cases, in the Chimpanzee, and even in the Siamang, whereas the pattern on the crowns, as it appears to me, does not really resemble that of any orang-utan molar. On the other hand, I confess not to feel at liberty to attribute, with VON KOENIGSWALD and others, some

EUG. DUBOIS: *The fossil human remains discovered in Java by Dr.* G. H. R. von Koenigswald *and attributed by him to* Pithecanthropus erectus, *in reality remains of* Homo sapiens soloensis. *Continuation.*

PLATE VI.



Brains, natural size, viewed from above, of a Polecat (*Putorius putorius*) (left figure) and a Stone Marten (*Martes foina*) (right figure), both animals full-grown; the species are of equal medium size of body and reckoned belonging to the same family.

EUG. DUBOIS: *The fossil human remains discovered in Java by Dr.* G. H. R. VON KOENIGSWALD *and attributed by him to* Pithecanthropus erectus, *in reality remains of* Homo sapiens soloensis. *Continuation.*

PLATE V.

a

c

(*a*) Upper jaw of a male Pithecanthropus (Sangiran, January 1939), viewed from the right side. *d*, diastema; J^1, alveolus of J^1; J^2, alveolus of J^2. nat. size.
(*c*) The same as (*a*) but viewed from in front. nat. size.

From "Nature" Dec. 2, 1939, Vol. 144, p. 927.
The figures one and a half times enlarged.

fossil molars they found in Java and China, to fossil orang-utan; the pattern on the crowns showing more resemblance to the proto-australians of Java and China.

The insufficiency of the endeavours of Weidenreich, von Koenigswald, and other able investigators, to prove in China and Java the reality of human evolution in Darwinian way, by gradual, slow and steady transformation of parts of the body, is a consequence of their neglecting the holismic or *Ganzheits* conception of the organisms, firmly established by the researches in experimental embryology, on which conception, although having their origin early and spontaneously in another starting-point, are also based my ~~own and~~ Louis Lapicque's researches on phylogenetic progress of the brain, especially the cerebrum, in the Vertebrates.

Concerning the principal result of these researches I may here refer to the annex Plate VI, and, further, to my paper "Die phylogenetische Grosshirnzunahme autonome Vervollkommnung der animalen Funktionen" [1]).

Obviously then, Man did not come into being in the Darwinian way, by gradual transformations required by the outer world power, but by inner world, autonomous, power. Nevertheless, Man is nearly allied to organisms even two autonomous degrees lower.

This — near relationship to the Anthropomorpha, living and extinct — is admirably proved by comparative anatomy and palæontology; however, those sciences were not able to teach us more concerning anthropogeny than their own insufficiency. I refer particularly to the important and excellent researches in South Africa. Much other meritorious endeavour, in the same way, has had no other results.

In finishing, I may call attention to the fact that the proto-australians we now know: Wadjak and Ngandong man, Peking man, also Rhodesian man, all of them, have the same absolute and relative low cranial capacity in common with the Australian aboriginal. The race has apparently neither progressed nor altered in this respect.

Furthermore, be it observed that, in accordance with the higher degree of autonomous brain progression, *Pithecanthropus erectus* possesses a calvaria-height-index equal to that of the small *Hylobates* species, not of the large *Symphalangus*, indeed the index of a *Hylobates agilis* being 35.5 and that of a *Symphalangus syndactylus* (both animals from the wild and full-grown) 24.5. The index of *Pithecanthropus erectus* was about 33.5.

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[1]) Biologia Generalis, Band 6, pp. 247—292. Wien und Leipzig 1930.

(To be continued.)

Physics. — *The mechanism of emulsion formation in turbulent flow.* I. Experimental Part. By P. H. CLAY. (Laboratorium voor Technische Physica van de Technische Hoogeschool te Delft.) (Communicated by Prof. J. M. BURGERS.)

(Communicated at the meeting of June 29, 1940.)

Zusammenfassung.

Der Mechanismus der mechanischen Bereitung von groben Emulsionen mit Hilfe der Wirkung eines turbulenten Strömungfeldes wird experimentell und theoretisch untersucht mit Rücksicht auf technische Anwendungen. Man kann zwei elementare Prozesse unterscheiden, die entgegengesetzte Wirkungen auf den Dispersionszustand ausüben: das Zerbrechen und das Zusammenfliessen der Tröpfchen. Die Wirkungen dieser Prozesse führen nach längerer Zeit zu einem Gleichgewicht im Dispersionszustand.

Zwei Apparate werden beschrieben, in denen die Tröpfchen der groben Emulsion im turbulenten Bewegungszustand, direkt bei sehr kurzer Belichtungszeit und Vergrösserung bis 40 ×, photographiert werden. Im ersten, technischen, Apparat wird die Turbulenz bei Strömung durch ein Rohr mit 10 cm Durchmesser benutzt; im zweiten Modelapparat die Turbulenz zwischen zwei coaxialen Zylindern, wobei der innere gedreht wird. Eine Methode zur Vermittlung der Grössenverteilung der Tröpfchen durch Ausmessen der photographischen Aufnahmen wird beschrieben. Messergebnisse werden gegeben. Es stellt sich heraus, dass die Tröpfchen auch während der heftigen Bewegung ihre Kugelform in sehr grosser Annäherung beibehalten.

1. *Introduction.*

In the normal procedure for the mechanical preparation of an emulsion, a coarse dispersion of the two fluids, which do not mix, is brought into a laminar or turbulent state of motion. The hydrodynamic field affects the state of dispersion of the emulsion in two different ways: a droplet of the dispersed phase may be broken up into two or more smaller droplets; on the other hand a droplet may meet another droplet with a certain chance of coalescence into one larger droplet. Our knowledge about the nature of these elementary processes and about the conditions under which they occur is still very poor, though we know much more about the physico-chemical properties of more or less stable emulsions at rest. We refer to: CLAYTON, *The theory of emulsions and their technical treatment* (Oxford 1935, Clarendon Press) p. 289.

The conditions under which these two elementary processes will occur depend on the properties of the hydrodynamic field used, as well as on various chemical and physical properties of the two fluids. The physical properties in question are the densities and viscosities of both liquid phases (ϱ and η for the exterior phase, ϱ' and η' for the dispersed phase), the volume percentage occupied by the dispersed phase (c %) and the interfacial tension (γ). The chemical properties will on the whole be the same as those considered in the theory of emulsions, consequently they concern chiefly the surface layer between the two liquids. Their influence will increase as the droplets become smaller. If the droplets are very small the electrical charge of the droplets may in special cases play a part. (The electrical conductivity of the exterior phase then has to fulfill certain special conditions.)

The state of dispersion which arises from the action of the elementary processes is usually characterized by the (differential) distribution of sizes of the droplets $n(a)da$ (in droplets per cm^3 emulsion); here a means the radius of the droplet if it were spherical. This state of dispersion is generally due to the action of the elementary processes in the mechanical mixer as well as to the original state of dispersion of the emulsion when it entered the mixer. When, however, a pair of liquids in a mixer is kept in the same state of motion long enough, the state of dispersion becomes stationary. Then the influences of the elementary processes which always counteract each other have led to an equilibrium in the state of dispersion. In this way it is possible that an emulsion which is not stable at rest, is stabilized by a state of motion. Such a stationary state of dispersion is free from the influence of the original state of dispersion at the entrance of the mixer.

In the technical applications of the mechanical preparation of emulsions, two possibilities are to be distinguished.

a. Sometimes the emulsion is the product wanted; then the emulsion has to meet some distinct requirements of stability etc. Generally the dispersion has to be sufficiently fine. In this case a is usually under 20 μ. The distribution of sizes of the droplets is in this case of some interest, but usually it is not of critical importance. During the mechanical preparation the elementary process of coalescence plays a secondary part; it is, however, of primary importance for the stability of the product and therefore it is studied amply, technically as well as from the purely scientific point of view.

b. In other cases the emulsion is but an intermediate stage in a certain technological physical or chemical process.

Two liquids, for instance, must be made to interact chemically or physically; one of the liquids is dispersed into the other in a mixing apparatus, usually by means of turbulent motion, untill a rather coarse emulsion is formed in which the large surface of contact promotes the interaction. After that the emulsion has to be separated into its components;

usually this is done in a settling tank or in a centrifuge. An example is the physical or chemical extraction process. In this case the mean size of the droplets of the temporary emulsion is of large technical and economical interest, because e.g. fine droplets promote the interaction, but impede the separation. With a given apparatus we may expect a rather sharp minimum in the total time required for interaction and separation together to occur at a certain critical mean size of the droplets. In this case both elementary processes play their part, while the emulsions remain so coarse that the physical properties of the liquids will have a preponderant influence. So far researches on this case are in an incipient stage only.

Here we give a report of the first results of some investigations which were projected in connection with the technical problems which arise in case *b*. They were carried out under the guidance of Professor W. J. D. VAN DIJCK in the Laboratory of Technical Physics of the Technical University in Delft. In two simple arrangements we studied the equilibrium in the state of dispersion mentioned above, in dependence of the physical properties of the liquids and of the properties of the field of flow. Moreover for some emulsions we considered the velocity with which this equilibrium was obtained, by producing a temporary preponderance of the process of breaking or a temporary preponderance of the process of coalescence. We chose such combinations of two liquids as are not or nearly not inclined to form a stable emulsion, and we considered the turbulent case only.

For the fields of flow we chose in the first place the turbulent field of flow through a 4″ circular tube at REYNOLDS numbers Re varying from 50.000 to 500.000, because this case of turbulence normally occurs in practice. Here, $Re = 2U_m R\varrho/\eta$, where U_m is the mean velocity of flow through the tube with radius R. The apparatus is described in No. 2. Besides we designed a model apparatus as described in No. 3, in order to have a check on the general validity of the results.

We studied the state of dispersion by direct photography of the emulsion in turbulent motion, as we wanted to be sure that we would obtain the undisturbed state of dispersion.

It was possible to take photographs because our emulsions remained rather coarse and because we restricted ourselves to small volume percentages of the dispersed phase (see No. 5).

The optical arrangement is described in No. 2. The photographs directly show us something about the elementary processes (see No. 6). By measuring the sharp images of the droplets on the photographs we could calculate the actual size distribution of the droplets. The procedure used is described in No. 5.

We give a discussion of the results of the experiments in No.'s 10 and 11, after a treatment of the elementary processes in No.'s 8 and 9. The properties of the turbulent field in our apparatus, which must be known for this treatment, are discussed in No. 7.

2. *The technical apparatus.*

In order to realize the stationary state of dispersion which is characteristic for turbulent flow through 4″ tube, we built a closed circuit of 4″ "linepipe" (see Fig. 1), in which the emulsion is kept in circulation by means of a centrifugal pump. The circuit consists of two straight parallel tubes, about

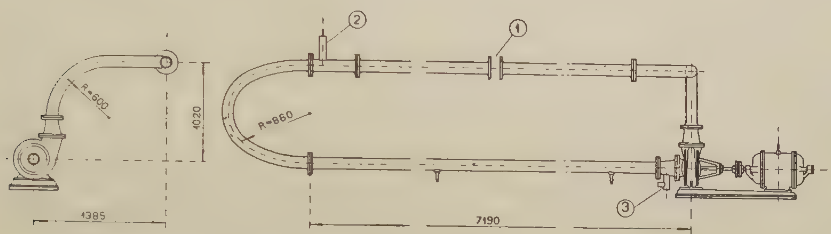

Fig. 1. The technical arrangement. At 1 the brass ring with the optical arrangement is mounted; 2 is the waste and de-aeration pipe; 3 is the inlet pipe. Hydrodynamic data for $Re = 4{,}5 \, . \, 10^4$ to $5{,}5 \, . \, 10^4$: the circuit consists of: 1376,5 cm straight tube of radius 5,1 cm and resistance coefficient $1{,}90 \, . \, 10^{-2}$; 453 cm curved tube of the same radius (radius of curvature 60 and 86 cm) and mean resistance coefficient $4{,}26 \, . \, 10^{-2}$. The radius of the rotor in the centrifugal pump is 22,5 cm; its circumferential velocity is $210 \; (U_m/100)^{1{,}04}$ cm/sec (U_m in cm/sec).

700 cm long, connected at one side by a piece of curved tube bent over 180°, with a radius of curvature of 86 cm. At the other end the lower tube is connected with the low pressure side of the centrifugal pump; the high pressure side of the pump is connected to the other end of the upper tube by means of two curved tubes of 90°, with a radius of curvature of 60 cm. The inner wall of the tubes remains fluent at all flange couplings of the circuit; the pump is so chosen, that the circumferential velocity of the rotor is lowest for the required supply and pressure drop, in order to disturb the emulsion as little as possible. Hydrodynamic data for the circuit are given below Fig. 1. From the resistance coefficient of $1{,}90 \, . \, 10^{-2}$ for the straight tubes in a very large range of Re we conclude that the grain of the wall is fine, but very inhomogeneous. In the curved tubes the average coefficient of resistance is $4{,}26 \, . \, 10^{-2}$. This much higher value is caused by the vortices which always are formed in curved tubes, as well as by a larger roughness of the wall. As the total length of the curved tubes is 453 cm, this high coefficient of resistance is of great importance for the emulsion formation in the circuit (see No. 9 and No. 10). The results of the experiments make it highly probable that the hydrodynamic field in the pump does not disturb the emulsifying properties of the circuit as caused by the turbulent flow through the tubes only (see No. 10).

The optical arrangement for photographing the droplets of the emulsion is mounted in a 10 cm broad brass ring, which is fitted in the middle of the straight tube at the output side of the pump (see 1, Fig. 1). As

the emulsion circulates very fast, there is no objection against taking photographs at this place in the circuit. Applying direct photography to the study of the emulsion droplets the following difficulties arose, which determined the choice of the arrangement.

a. The emulsion is rather untransparent.

b. The magnification should be sufficiently high, while the object is moving very fast. This requires a very short time of illumination, which gives difficulties with the intensity. If we wish our photographs sharp within 0,1 mm at a magnification 50 and if the mean velocity of flow is 500 cm/sec ($Re \approx$ 500.000 in water) the time of illumination should be less then 2.10^{-7} sec.

c. The layer of emulsion droplets between the focussed layer and the object-glass should be thin, in order not to disturb the image formation (see No. 5).

d. The field of flow in the neighbourhood of the photographed droplets and also the general flow should be disturbed as little as possible.

The arrangement represented in Fig. 2 had the following characteristics.

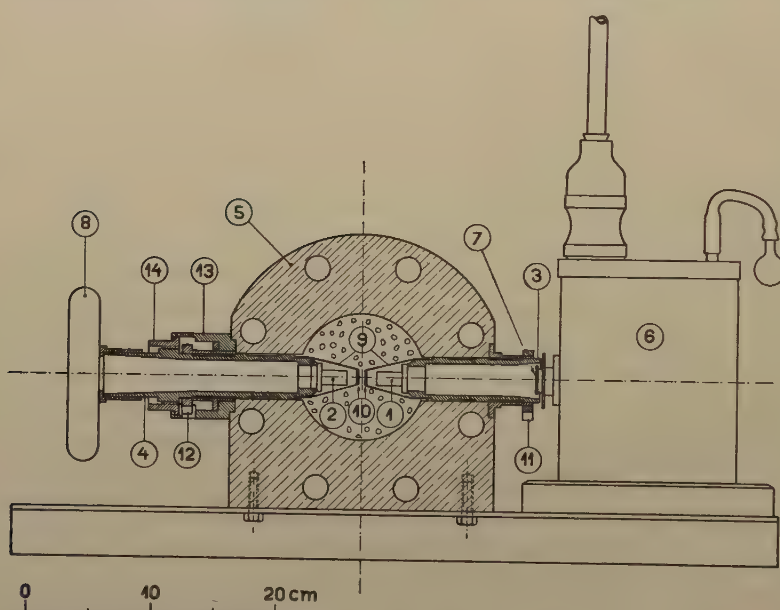

Fig. 2. A section through the brass ring with the optical arrangement in the circuit, as described in No. 2. The tubes 3 and 4 which pass water tight through ring 5, contain the object-glasses 1 and 2. They are made of stainless steel; their positions are secured by means of the clamping rings 11 and 12. Tube 4 can be adjusted with the aid of the threaded ring 14.

The emulsion in the centre of the tube is photographed with transmitted illumination. The difference in the refractive indices of the two liquid phases must produce the necessary contrast. Object-glass 2 and condensor 1 (see Fig. 2), both of Zeiss type *A*, are placed as in an ordinary

microscope. The image formed by the object-glass falls directly on the sensitive material of a camera 8, in order to be able to come out with a low intensity of illumination (Leica camera with Agfa FK isochrome film).

In connection with the points a and c the distance between the condenser and the object-glass is chosen so small as is consistent with point d. To this purpose the dry object-glasses 1 and 2 which are used as object-glass and condenser are roofed by little waterproof caps 9 with small plan parallel glass windows 10. The emulsion flows around these caps and through the slit of about 4 mm width between the windows, where it is illuminated through the condenser window; the droplets in the middle of the slit are focussed through the object-glass window on the sensitive material. Thus an emulsion layer of only about 2 mm thickness disturbs the illumination and another of the same thickness disturbs the image formation. The direct magnification is 12,6; the negatives are magnified in printing. The highest total magnification is 100.

The apparatus for the illumination 6, Fig. 2, consists of a multiplate condenser of about 500 $\mu\mu F$, with a very small inner selfinduction. Directly attached to it is a spark gap consisting of two tapered magnesium electrodes at a distance of 1,5 à 2 mm. The whole is placed in limpid transformer oil; the spark passes at 15 à 20 kV. The circuit, formed by the condenser and the spark gap, has a calculated resonance frequency of 3.10^7. An image of the spark is formed by a magnifying glass 7 in the first focal plane of the condenser. Always the photographs were sufficiently clear and sharp provided the focussing was right. Fig. 3 gives a photograph of the optical arrangement; it also shows an influence machine of Hommelsdorf used as the source of tension for the spark.

3. *The model arrangement.*

The second apparatus was designed for orientating investigations. For the turbulent field we chose the field of flow in the space between two coaxial cylinders with radii R_1 and R_2 ($R_1 > R_2$), the inner one rotating at an angular velocity ω. This arrangement is simple, and disturbances as caused by the pump and by the curved parts in the circuit do not occur.

As the dimensions of the model are small, an optical arrangement which protrudes in the emulsion, as is used in the circuit, would disturb the flow too much.

For this reason we have chosen the arrangement shown in Fig. 4. A nickle plated brass cylinder 2 rotates with angular velocity ω in a cylindrical glass dish 1. The dish 1 is covered and closed by means of glass plate 4 and a cork packing ring 3. The (annular) space between the dish 1 and the cylinder 2 is occupied by the emulsion. The axis of the cylinder 2 is led through plate 4 by means of a liquid trap 5 and is driven by the gears 9. The illuminating light passes axially through the whole system. For the rest the optical arrangement is similar to the arrangement described in No. 2. The layer which is sharply in focus lies about 5 mm

below the glass plate in the emulsion. The same spark gap as in No. 2 is used as source of illumination (15 on Fig. 4). Of course the illuminating

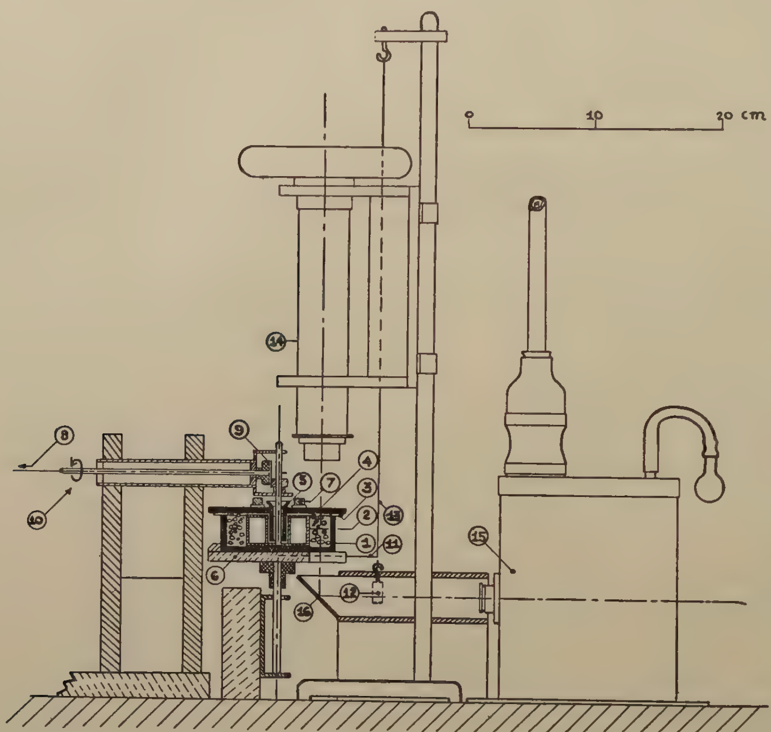

Fig. 4. A section through the model arrangement as described in No. 3. The couple measurements are made with the aid of the threads 11 and 13 and the weight 12; 3 is a cork packing ring; the small strips 7 produce a pressure upon the glass plate; 16 is a mirror.

light rays are rather badly scattered and weakened by absorption before they reach the focal plane in this optical arrangement. Therefore the magnification must be lower than in the circuit (the direct magnification was 3,44; the highest total magnification was 20). For the same reason we are more restricted in the volume percentage, occupied by the dispersed phase in the model (see No. 5), than is the case in the circuit.

Our knowledge about the field of flow between long coaxial rotating cylinders is extensively treated in a recent compilatory work of S. GOLDSTEIN[1]. There it is proved that the field of flow may be unstable. When the REYNOLDS number, defined by $Re = \varrho \omega R_1 (R_1 - R_2)/\eta$ rises above a certain critical value, dependent on $(R_1 - R_2)/R_1$, annular vortices appear.

In the model the axial length is limited and this probably has some influence on the field of flow, which also may be disturbed by the slits at the bottom of the disk, at the glass plate and in the liquid trap. The results obtained with the model, which are discussed in No. 10, contain no

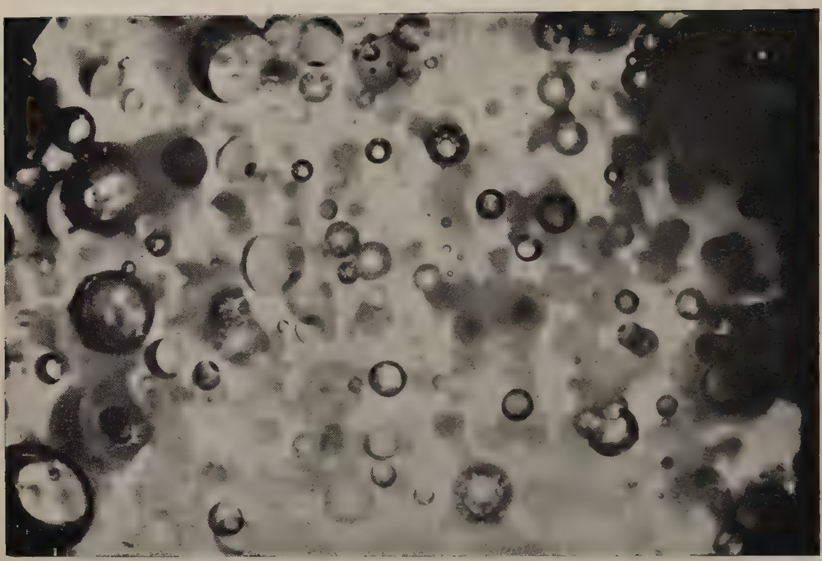

Fig. 8. Photograph (16,3) series *R* in the model. Magnification 10. Many droplets are seen clinging together.

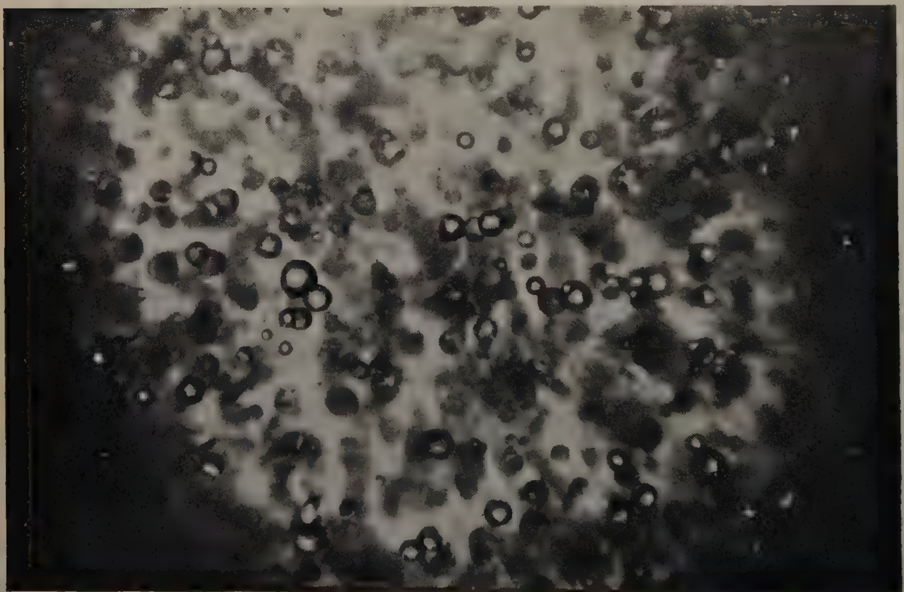

Fig. 9. Photograph (19,7) series *T* in the circuit. Magnification 40.

Fig. 3. The optical arrangement in the circuit, seen from the camera side.

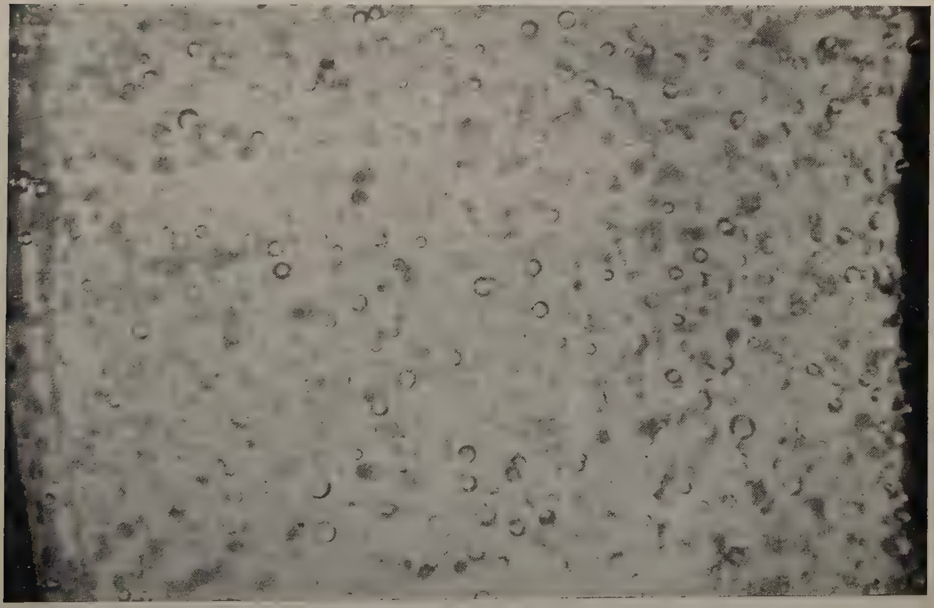

Fig. 7. Photograph (1,18) series *M* in the model. Magnification 7,7; the distance between the place where u_S/U_τ has reached its maximum value and the inner wall is about 1,5 cm on the picture.

evidence for such disturbing influences, however. We always had two annular vortices.

In order to obtain some information about the hydrodynamic field in this model arrangement, we measured the couple which the fluid exerts on the dish. For this purpose the dish is placed on the little table 6 (fig. 4), which can rotate freely around the common vertical axis of the cylinders. If we neglect the couples exerted on the bottom *), on the glass plate and in the liquid trap, we can calculate the mean shearing stress at the walls of the cylinders (τ_1 at R_1; $\tau_2 = (R_1/R_2)\ \tau_1$ at R_2) and derive a resistance coefficient $\tau_1/\varrho\omega^2 R_1{}^2$. The result gives Fig. 5, together with

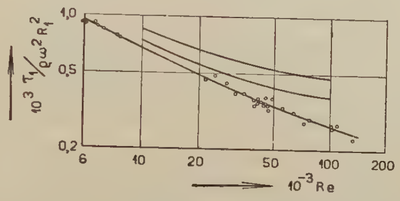

Fig. 5. The curves show the resistance coefficient $\tau_1/\varrho\omega^2 R_1{}^2$ as a function of the REYNOLDS number $Re = \varrho\omega R_1 (R_1 - R_2)/\eta$ for the flow between coaxial cylinders with the inner cylinder rotating.

Curve 1 for the model arrangement: $(R_1 - R_2)/R_1 = 0{,}40$
Curve 2 for long cylinders : $(R_1 - R_2)/R_1 = 0{,}21$
Curve 3 for long cylinders : $(R_1 - R_2)/R_1 = 0{,}15$

The curves 2 and 3 are given by GOLDSTEIN; the data are taken from TAYLOR.

some curves calculated from measurements of G. I. TAYLOR at other values of $(R_1 - R_2)/R_1$. A comparison of the three curves justifies our neglections sufficiently.

4. *The experiments.*

The emulsions must be homogeneous, separation by settling must be prevented.

This gives no difficulties in the circuit where the turbulence is intense, where the dispersion is fine and where the turbulent diffusion is very active. In the model arrangement the annular vortices take care of the mixing if the turbulent diffusion may be deficient. In executing the experiments, groups of 3 to 6 photographs were taken of the emulsions under different selected circumstances; this we call a "measurement". By measuring the sharp images of droplets on the photographs (see No. 5) we find the size distribution of the emulsion droplets. The photographs of one measurement were taken with a difference in time of 5 to 20 seconds.

*) We obtained an estimation of the couple exerted on the bottom of the dish only by filling the dish to a height of but 3 mm above the bottom. The couple measured in this case was always less than 10 % of the couple measured in the normal case, even at the lowest REYNOLDS numbers.

We call the measurements made in succession with one combination of liquids a "series" (indicated it by a capital letter).

The following types of measurements were made:

1. Measurements with an emulsion in equilibrium, i.e. an emulsion which was kept in the same state of motion during 20 to 60 minutes. (This period appears usually to be long enough; see the data and also No. 11).

2. Beginning with a measured emulsion, in equilibrium at $Re = Re_1$, we suddenly changed Re_1 into Re_2 and made a measurement after an interval of time T. If $Re_2 > Re_1$, the breaking process has predominated during the time T, and a comparison of the original state of dispersion with the new one gives an impression of the activity of this process of breaking at $Re = Re_2$ during the time T. If $Re_2 < Re_1$, we obtain an impression of the process of coalescence (see No. 11).

The measurements made with the model arrangement are given in table A, those made with the circuit are given in table B.

The liquids which we used in the model were acquired from a pharmaceutical chemist; besides we used distilled water and very pure kerosene. During the model measurements the temperature always remained sufficiently constant (about 18° C).

In the circuit we used commercial liquids and tapwater. The liquids were polluted somewhat during a series of measurements because rust particles etc. left the wall. The temperature always rose gradually during the execution of the measurements of one series which was due to the energy dissipation in the circulating emulsion. We have corrected the properties of the liquid phases for the change of temperature.

The number of measurements made in the circuit is still small. In the model arrangement we made measurements at $\varrho = \varrho'$ with η from 0,6 cP till 10 cP, η'/η from 0,3 till 30, γ from 1 dyne/cm till 49 dyne/cm, c till 4 % and Re from 5000 till 100.000; and some measurements at $\varrho' > \varrho$.

5. *The determination of the size distributions.*

The photographs show images of droplets in any degree of sharpness. This makes the measuring out of the photographs more difficult. However, it is a general and remarkable phenomenon that the sharp images of droplets on the photographs make it certain, that nearly all droplets in the turbulent emulsion retain the spherical form notwithstanding the violent motion. This fact (see No. 6) makes it possible to measure out the photographs on a reasonable basis, because the sharpness of the images of spherical particles obeys sufficiently simple relations.

If we put a lower limit to the sharpness of the images of the spherical droplets in the photographs of one measurement, we restrict ourselves to droplets with their centres in a certain region or layer of sharp image formation. This layer will be limited in first approximation by two parallel planes, perpendicular to the optical axis on both sides of the focussed plane, at a mutual distance which we call the depth of focus s. In a given optical system s may depend on the refractive indices of the droplets and of the exterior phase, and also on the droplet radius. We cannot predict this dependence theoretically.

From the size distribution of the droplets with sharp images on the photographs of

one measurement, we can derive the real size distribution of the droplets in the emulsion, provided we know s as a function of a for the case in question. The applicability of this procedure depends on the reproducibility of the criterion of sharpness and on the fact whether we can measure s as a function of a.

The criterion of sharpness must be a subjective one. We chose the subjectively judged image sharpness of the circumference of the droplets on the photographs and measured $s = s\,(a)$ with the aid of the following indirect method.

In an emulsion with c % of the volume occupied by the dispersed phase and with $n(a)\,da$ droplets per cm^3 with a radius between a and $a + da$, the following relation must be fulfilled:

$$\int_0^\infty n\,(a)\,.\,{}^4/_3\,\pi\,a^3\,da = \frac{c}{100}.$$

If we measure out a surface O on the photographs at a magnification V, we obtain a size distribution of the droplets in the region of sharp image formation given by $n'(a)\,da$.

Now $n\,(a) = n'\,(a)\,\dfrac{V^2}{O\,s\,(a)}$, and therefore $$\frac{V^2}{O}\int_0^\infty \frac{n'\,(a)}{s\,(a)}\,.\,{}^4/_3\,\pi\,a^3\,da = \frac{c}{100}. \qquad (5,1)$$

It appears that in the emulsions investigated the huge part of the dispersed volume is occupied by droplets of a limited size region which may be characterized by a radius a_1. (see No. 6, remark *b1*). Therefore (5,1) can be approximated by:

$$\frac{V^2}{O\,s\,(a_1)}\int_0^\infty n'\,(a)\,.\,{}^4/_3\,\pi\,a^3\,da = \frac{c}{100}.$$

From this equation we can calculate $s(a_1)$. Using different measurements of one series we obtain s as a function of a for that series. The method can be refined a little bit, but we have to extrapolate to the largest and to the smallest droplets. In practice we work with size classes for the droplets (see Figs. 10, 11 and 12).

All photographs have been measured out by the same person and for him we determined the regions of sharp image formation. In the circuit we found the same relation between s and a with both emulsions; consequently the refractive indices had no noticeable influence. With the model arrangement the result is the same, but here c appears to have some influence on the depth of focus s, what may be explained by the primitive illuminating optics. The results for the circuit and for the model (at low values of c) are given in Fig. 6.

We cannot expect a large accuracy in this procedure: the clear photographs fulfilled (5,1) with a maximum deviation of 20 %. Per measurement we had a number of sharp images of droplets varying from 100 to 300. This result is connected with the form of the distribution curves (see No. 6, remark *b* 1).

The sharp images of the droplets covered the surface of the photographs for 14 % at most. Finally we remark that with the optical system of the circuit $s(a)/a$ lies between 1 and 2. With that of the model arrangement it lies between 3 and 10, as can be deduced from Fig. 6.

The region of applicability of the photographic method is limited by the disturbance of the image formation by the droplets of the emulsion layer between the layer in focus and the object glass. This disturbance depends on the optical system used and on the refractive indices. In first approximation it will be proportional to the droplet surface

per cm³ ($\sim c/as$ if as^2 is used to denote the average value of a^2), and proportional to the thickness of the disturbing layer.

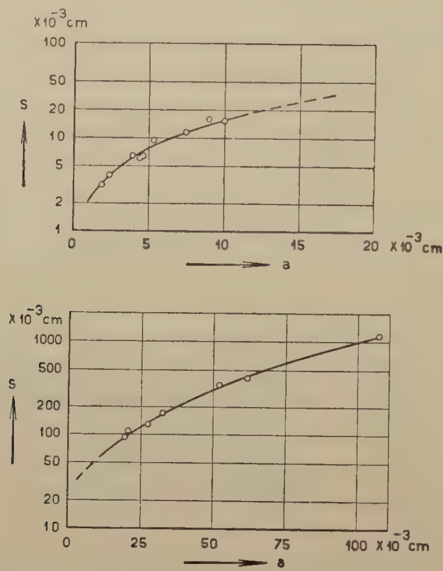

Fig. 6. The depth of focus s as a function of the radius a of the droplet for the technical arrangement (upper curve, data from series S, T, U and V) and for the model arrangement (lower curve, data from series H and K).

6. *Results. General remarks.*

The result of the investigations is embodied in photographs and in size distributions of the states of dispersion in the investigated emulsions.

We first consider the photographs (see Figs. 7, 8 and 9). The model photographs show a section through the whole turbulent field (see No. 9). The circuit photographs give the situation in the centre of the tube only, which may be disturbed moreover by the optical arrangement. Nevertheless all the photographs give the same general impression, in which the following pecularities call for attention.

*a*1. Nearly all the droplets are spherical, also those near to the wall in the model.

*a*2. Breaking processes are seen only seldom. Some droplets show local deformations in the form of tiny protuberances, which probably will ultimately lead to the separation of a very small droplet. These details on the photographs could not be reproduced.

*a*3. On the photographs we often see droplets which apparently cling together. Especially we often see that one or more very small droplets adhere to one larger droplet (see Fig. 8). Certain emulsions show this phenomenon very often (see No. 8).

The small number of breaking processes indicates that these must have a very rapid course.

Some distributions of sizes are given in Figs. 10, 11 and 12.

The distributions of sizes can be treated in two different ways. We can judge the qualitatively according to their general form and compare them.

Then the following points are observed:

*b*1. All curves decline steeply towards the side of the larger droplets (see No. 5). This is in contrast with the distributions of sizes, measured

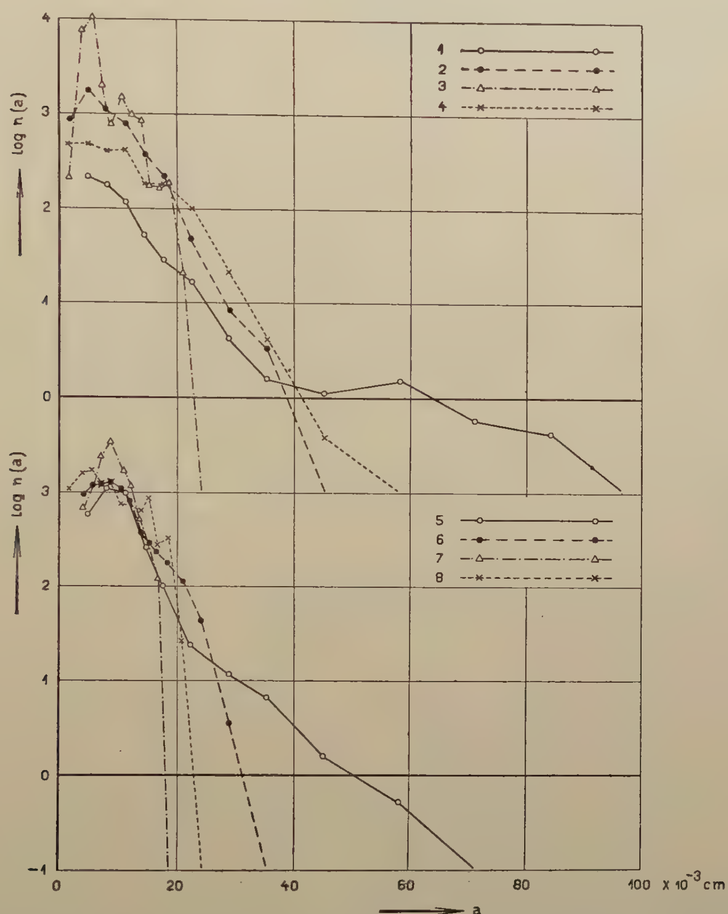

Fig. 10. Size frequency curves of the droplets of some emulsions made in the model:
Curve (1) is (4—6,9) series *F*; (2) is (7—8,9) series *F*; (3) is (1—2,10) series *F*; (4) is (9—10,18) series *N*; (5) is (11—12,13) series *G*; (6) is (11—12,14) series *I*; (7) is (5—6,15) series *J*; (8) is (5—6,16) series *K*. The scale for $n(a)$ is logarithmic*).

*) The logarithmic scale for $n(a)$ in Figs. 10 and 11 is taken in such a way, that *log* $n(a) = 0$ corresponds to $n(a)\,da = 0{,}118$ droplets per cm^3 per vol. % of the dispersed phase, if da is measured in 10^{-3} cm. In Fig. 12 in the same way $n(a)\,da = 127$ droplets per cm^3 per vol. % of the dispersed phase. This allows direct comparison of the various curves.

with rain drops or fog drops, and with older emulsions (which were not homogenized very rigorously), and in which during a long time only

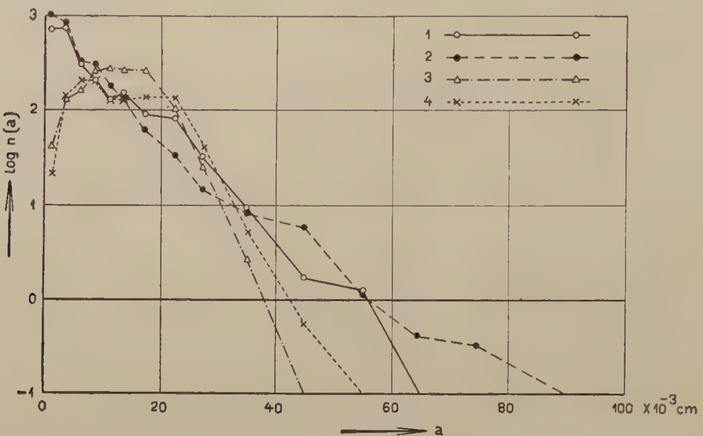

Fig. 11. Size frequency curves of the droplets of some emulsions made in the model. Curve (1) is (16—20,2) series O; (2) is (22—26,2) series O; (3) is (0—5,3) series Q; (4) is (8—12,5) series Q. The scale for $n(a)$ is logarithmic*).

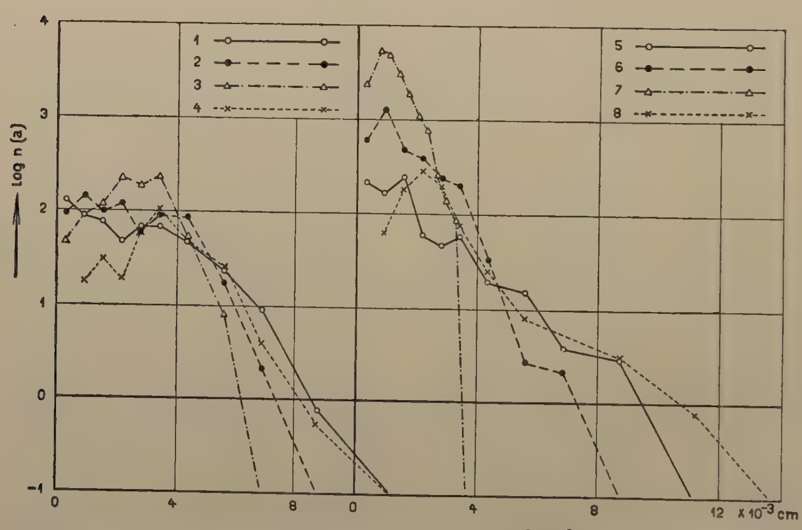

Fig. 12. Size frequency curves of the droplets of some emulsions made in the circuit. Curve (1) is (45—4,7) series T; (2) is (6—10,7) series T; (3) is (21—25,7) series T; (4) is (28—31,7) series T; (5) is (45—7,9) series U; (6) is (12—15,9) series U; (7) is (16—18,9) series U; (8) is (32—24,9) series V. The scale for $n(a)$ is logarithmic*).

coalescence was in action. So the steep decline is characteristic for the action of the breaking process (see No.'s 9 and 10).

b2. To the side of the smaller droplets the size distributions show

*) See note p. 863.

various forms: some are flat, some rise, other ones have a distinct maximum at a certain value of a.

c. We can also consider the states of dispersion on a quantitative basis. To that purpose from the size distribution of the emulsion droplets we calculate the integral distributions of the dispersed volume, and from these we take some characteristic radii: a_{95}, a_{50} and a_{10}. The droplets with $a \leqslant a_{95}$ contain 95 % of the dispersed volume, those with $a \leqslant a_{50}$ contain 50 % and those with $a \leqslant a_{10}$ contain 10 %. In connection with $b1$ we may consider a_{95} as the radius of the largest droplets in our emulsions. The tables give the values of a_{95}, a_{95}/a_{50} and a_{10}/a_{50} for all the emulsions investigated.

From the values of the last quantities, it follows that the integral distributions of the dispersed volume are usually nearly the same for the greater part of the emulsions. These quantitative data are very important especially in technical applications.

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Mathematics. — *Ueber affine Invarianten bei quadratischen Formen.* Von O. BOTTEMA. (Communicated by Prof. R. WEITZENBÖCK.)

(Communicated at the meeting of June 29, 1940.)

1. In zwei in diesen Proceedings erschienenen Arbeiten hat WEITZENBÖCK [1]) die Invarianten einer quadratischen Form und einer Linearform bei der speziellen affinen Gruppe bzw. in der Ebene und in R_3 untersucht und einige geometrische Anwendungen gegeben.

Wir machen im Folgenden einige Bemerkungen über die von WEITZENBÖCK gebildeten absoluten Invarianten und über die von ihm für $n=2$ und $n=3$ gelöste geometrische Frage, welche wir für allgemeines n erledigen.

Wir beschränken uns dabei vorläufig auf den Fall der Ebene. WEITZENBÖCK zeigt dass eine ternäre quadratische Form und eine lineare Form in homogenen Punktkoordinaten $x_1 : x_2 : x_3$,

$$f = \Sigma\, a_{ik}\, x_i\, x_k \;,\; (v'\, x) = \Sigma\, v'_i\, x_i \;\; \ldots \ldots \quad (1)$$

ein kleinstes volles System von ganz-rationalen affinen Invarianten besitzen das aus fünf Bildungen besteht, welche nicht-symbolisch geschrieben folgendermassen aussehen:

$$\left.\begin{aligned}
&D = \begin{vmatrix} a_{11} & a_{12} & a_{12} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad\qquad C = a_{11}\, a_{22} - a_{12}^2 \\
&\Phi = - \begin{vmatrix} a_{11} & a_{12} & a_{13} & v'_1 \\ a_{21} & a_{22} & a_{23} & v'_2 \\ a_{31} & a_{32} & a_{33} & v'_3 \\ v'_1 & v'_2 & v'_3 & 0 \end{vmatrix} \quad U = a_{11}\, v_1'^2 - 2\, a_{12}\, v'_1\, v'_2 + a_{22}\, v_2'^2 \\
&M = v'_1\,(a_{12}\, a_{23} - a_{13}\, a_{22}) + v'_2\,(a_{13}\, a_{12} - a_{11}\, a_{23}) + v'_3\,(a_{11}\, a_{22} - a_{12}^2).
\end{aligned}\right\} \quad . \quad (2)$$

Zwischen diesen Komitanten besteht eine einzige Syzygie

$$C\,\Phi = D\,U + M^2. \;\; \ldots \ldots \quad (3)$$

Auf Grund dieser Gleichung wird nun bei der Bestimmung der absoluten Invarianten D fortgelassen. WEITZENBÖCK bildet die absoluten Invarianten

$$\alpha = \frac{M^2}{C\,\Phi} \;,\; \beta = \frac{C\,U^2}{\Phi^2} \;\; \ldots \ldots \quad (4)$$

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[1]) WEITZENBÖCK, Ueber affine Invarianten bei Kegelschnitten, Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **43**, 159—167; Zur Affingeometrie der F_2 in R_3, 168—178, (1940).

und gibt eine geometrische Deutung der Invarianten R und S, welchen den Bedingungen

$$R^2 = \frac{-1}{\alpha\beta} \quad , \quad S^2 = \frac{-\alpha}{\beta} \quad \ldots\ldots\ldots \quad (5)$$

genügen.

2. Wir möchten nun den Invarianten α und β ein anderes Paar von absoluten Invarianten gegenüberstellen, durch die ebenfalls alle weiteren absoluten Invarianten ausdrückbar sind und welche invariantentheoretisch einen gewissen Vorzug haben.

Wenn wir statt D die Komitante U ausser Acht lassen, so gibt der Ansatz

$$J = D^{a_1} C^{a_2} \Phi^{a_3} M^{a_4}$$

da J vom Grade Null in den Koeffizienten a_{ik} und in den Linienkoordinaten v'_i sein muss, für die Exponenten die Gleichungen

$$3a_1 + 2a_2 + 2a_3 + 2a_4 = 0 \quad , \quad 2a_3 + a_4 = 0$$

woraus

$$a_4 = -2a_3 \quad , \quad a_2 = -\tfrac{3}{2} a_1 + a_3$$

also

$$J = (D C^{-\frac{3}{2}})^{a_1} (C \Phi M^{-2})^{a_3}$$

folgt.

Es sind also

$$k = D^2 C^{-3} \quad , \quad l = C \Phi M^{-2} \quad \ldots\ldots\ldots \quad (6)$$

ebenfalls zwei unabhängige rationale absolute Invarianten. Den Zusammenhang mit α und β zeigen folgende Gleichungen:

$$k = \frac{(1-\alpha)^2}{\beta} \quad , \quad l = \frac{1}{\alpha} \quad \ldots\ldots\ldots \quad (7)$$

und

$$\alpha = \frac{1}{l} \quad , \quad \beta = \frac{(l-1)^2}{k l^2} \quad \ldots\ldots\ldots \quad (8)$$

Die Invarianten k bzw. l haben nun folgende Eigenschaften: die für das aus einer quadratischen Form und einer Linearform gebildete System

abgeleitete absolute Invariante k der speziellen affinen Gruppe, ist auch eine absolute Invariante für die quadratische Form allein; die für das aus einer quadratischen Form und einer Linearform gebildete System abgeleitete absolute Invariante l der speziellen affinen Gruppe, ist auch eine absolute Invariante der allgemeinen affinen Gruppe.

Die erste Eigenschaft folgt unmittelbar aus der Definition von k; in D^2C^{-3} treten nur die Koeffizienten der quadratischen Form auf.

Die zweite Eigenschaft geht hervor aus der Bemerkung, dass C, Φ und M Invarianten der allgemeinen affinen Gruppe sind und je das Gewicht 2 haben; $C\Phi M^{-2}$ ist also eine absolute Invariante dieser Gruppe. Man zeigt leicht, dass sie die einzige absolute Invariante ist.

Was die WEITZENBÖCKschen Invarianten betrifft, so ist offenbar α eine absolute Invariante gegenüber der allgemeinen affinen Gruppe, während β, R und S diese Eigenschaft nicht zukommt.

Von den Invarianten R und S hat WEITZENBÖCK eine geometrische Deutung gegeben. Wir geben eine solche von k und l (und also von α).

Was k betrifft, die eine absolute, speziell-affine Invariante des durch $f=0$ dargestellten Kegelschnittes ist, so zeigt man leicht:

$$k=\varrho^3 \quad \dots\dots\dots\dots \quad (9)$$

wo ϱ der affine Krümmungshalbmesser des Kegelschnittes ist, also der reziproke Wert der Affinkrümmung [2]). Für $k>0$, $k=0$, $k<0$ ist der Kegelschnitt bzw. eine Ellipse, eine Parabel, eine Hyperbel. Im ersten Fall hat man z.B. noch die einfache Deutung [3])

$$k=\left(\frac{F}{\pi}\right)^2 \quad \dots\dots\dots\dots \quad (10)$$

wo F den Flächeninhalt der Ellipse bedeutet; auch bei der Hyperbel könnte man eine analoge Bedeutung angeben.

Was die zweite Invariante l betrifft, wollen wir eine geometrische Bedeutung angeben nicht von l selbst, sondern von dem Ausdruck

$$l'=\frac{l}{l-1}=\frac{C\Phi}{C\Phi-M^2}=\frac{C\Phi}{DU} \quad \dots\dots\dots \quad (11)$$

welche von WEITZENBÖCK in der 2. Arbeit genannt und mit δ bezeichnet worden ist (S. 170). Wir gehen dabei folgendermassen vor.

Die Schnittpunkte P und Q des Kegelschnittes $f=0$ und der Geraden

$$v'_1 x_1 + v'_2 x_2 + v'_3 x_3 = 0$$

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2) BLASCHKE, Differentialgeometrie, II, S. 18 (1923).
3) BLASCHKE, l.c. S. 18.

haben bzw. die Koordinaten (p_1, p_2, p_3) und (q_1, q_2, q_3). Wenn wir aus $f = 0$ und $(v'x) = 0$ die Koordinate x_2 mittels

$$x_2 = \frac{-v_1' x_1 - v_3' x_3}{v_2'}$$

eliminieren, so entsteht eine quadratische Gleichung für $x = \frac{x_1}{x_3}$, welche so aussieht:

$$\left.\begin{array}{r} U x^2 + 2x(-a_{23} v_1' v_2' + a_{22} v_1' v_3' + a_{13} v_2'^2 - a_{12} v_2' v_3') + \\ + (a_{33} v_2' - 2 a_{23} v_2' v_3' + a_{22} v_3'^2) = 0. \end{array}\right\} \quad . \qquad (12)$$

Die Wurzeln dieser Gleichung sind offenbar $\frac{p_1}{p_3}$ und $\frac{q_1}{q_3}$. Nun ist die Diskriminante $\triangle$ dieser Gleichung Null, wenn die Gerade $(v'x) = 0$ den Kegelschnitt berührt, das heisst wenn $\Phi = 0$; Φ ist also ein Faktor von $\triangle$ und man findet leicht

$$\triangle = - v_2'^2 \Phi. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

Für das Quadrat der Wurzeldifferenz hat man also:

$$\left(\frac{p_1}{p_3} - \frac{q_1}{q_3}\right)^2 = \frac{-4 v_2'^2 \Phi}{U^2}. \quad . \; . \; . \; . \; . \; . \; . \quad (14)$$

Wir betrachten jetzt die mit $(v'x) = 0$ parallele Gerade, welche durch den Mittelpunkt des Kegelschnittes geht. Sie hat die Gleichung

$$v_1' x_1 + v_2' x_2 + v_3' (1 + p) x_3 = 0. \quad . \; . \; . \; . \; . \; . \quad (15)$$

Die Schnittpunkte dieser Geraden mit dem Kegelschnitt werden mit $A \equiv (a_1, a_2, a_3)$ und $B \equiv (b_1, b_2, b_3)$ bezeichnet. Man hat also

$$\left(\frac{a_1}{a_3} - \frac{b_1}{b_3}\right)^2 = \frac{-4 v_2'^2 \Phi_1}{U_1^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

wenn Φ_1 und U_1 die Funktionen sind, welche man erhält indem man in Φ und U die Variabele v_3' durch $(1 + p) v_3'$ ersetzt. Diese Substitution gibt uns aber:

$$U_1 = U \quad , \quad \Phi_1 = \Phi + 2 p v_3' M + p^2 v_3' C \quad . \; . \; . \; . \quad (17)$$

während die Bedingung, dasz die parallele Gerade durch den Mittelpunkt gehe die Gleichung

$$M + p v_3' C = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

ergibt. Wir haben also

$$\Phi_1 = \Phi - \frac{M^2}{C} = \frac{DU}{C} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

Aus (14), (16) und (19) geht nun hervor:

$$\frac{\left(\frac{p_1}{p_3}-\frac{q_1}{q_3}\right)^2}{\left(\frac{a_1}{a_3}-\frac{b_1}{b_3}\right)^2}=\frac{\Phi}{\Phi_1}=\frac{C\Phi}{DU}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Wir finden somit:

Sind P und Q die Schnittpunkte der Geraden $(v'x)=0$ und des Kegelschnittes $f=0$, A und B die Schnittpunkte des Kegelschnittes und der mit $(v'x)=0$ parallelen Geraden durch den Mittelpunkt des Kegelschnittes, so ist

$$l'=\left(\frac{PQ}{AB}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Das Quadrat eines Streckenverhältnisses auf parallelen (oder zusammenfallenden) Geraden ist augenscheinlich eine absolute Invariante für die Gruppe der allgemeinen Affinitäten. Die abgeleiteten Eigenschaften sind offenbar gültig für jeden Mittelpunktskegelschnitt.

3. Ist $f=0$ eine reelle Ellipse, die von der Geraden $(v'x)=0$ in zwei reellen Punkten geschnitten wird, so zerteilt die Gerade die Ellipse in zwei Segmente. WEITZENBÖCK hat gezeigt, wie sich die Flächeninhalte F' und F'' in den absoluten affinen Invarianten ausdrücken lassen. Die Berechnung kann man vereinfachen, indem man die Figur auf ein geeignet gewähltes Koordinatensystem bezieht.

Wird der Ursprung im Mittelpunkt der Ellipse genommen, die x_1-Achse mit der Geraden $(v'x)=0$ parallel und ist die x_2-Achse im Bezug auf die Ellipse mit der x_1-Achse konjugiert, so erhalten die Ellipse und die Gerade, nachdem man noch eine geeignete speziell-affine Transformation ausführt, die Gleichungen:

$$x_1^2+x_2^2-R^2x_3^2=0 \quad , \quad x_2-p\,x_3=0. \quad . \quad . \quad . \quad . \quad (22)$$

Man hat dann:

$$D=-R^2 \ , \ C=1 \ , \ \Phi=-(R^2-p^2) \ , \ M=-p \ , \ U=1$$

und also:

$$k=R^4 \ , \ l=\frac{p^2-R^2}{p^2} \ , \ l'=\frac{R^2-p^2}{R^2}$$

womit die angegebene Deutung der Invariante l' bestätigt wird. Führte man eine euklidische Massbestimmung ein, wobei die isotropen Punkte in den uneigentlichen Punkten der Ellipse gewählt werden, so hätte man

$$l'=\sin^2\alpha \ , \ l=-tg^2\alpha$$

wo 2α einer der Bogen PQ der nun zum Kreis gewordenen Ellipse ist. Die genannten Flächeninhalte kann man jetzt elementargeometrisch bestimmen. Für das (allgemein affin-invariante) Verhältnis dieser Zahlen findet man:

$$\left.\begin{aligned} F' : F'' &= \left(\frac{\pi}{2} - \operatorname{arc\,cotg} \sqrt{-l} - \frac{\sqrt{-l}}{l-1}\right) : \left(\frac{\pi}{2} + \operatorname{arc\,cotg} \sqrt{-l} + \frac{\sqrt{-l}}{l-1}\right) \\ &= \left(\frac{\pi}{2} - \arccos \sqrt{l'} - \sqrt{l'(1-l')}\right) : \left(\frac{\pi}{2} + \arccos \sqrt{l'} + \sqrt{l'(1-l')}\right) \end{aligned}\right\} \quad (23)$$

während

$$F' + F'' = \pi \sqrt{k} \quad \ldots\ldots\ldots\ldots \quad (24)$$

Mittels (5) und (8) werden die WEITZENBÖCKschen Resultate zurückgewonnen [4]. Auch die für die Parabel und die Hyperbel gegebenen Formeln können mit der Methode des kanonischen Koordinatensystems abgeleitet werden.

4. Die oben gewonnenen Eigenschaften kann man auf den Raum von n Dimensionen übertragen. In R_n haben eine quadratische Form $f = \sum\limits_{i,k=1}^{n+1} a_{ik} x_i x_k$ und eine Linearform $(v'x) = \sum\limits_{i=1}^{n+1} v'_i x_i$ folgende fünf Komitanten:

$$\left.\begin{aligned} D &= \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1,n+1} \\ a_{21} & a_{22} & \ldots & a_{2,n+1} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n+1,1} & a_{n+1,2} & \ldots & a_{n+1,n+1} \end{vmatrix}, \quad C = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix} \\ \Phi &= -\begin{vmatrix} a_{11} & \ldots & a_{1,n+1} & v'_1 \\ \cdot & \cdot & \cdot & v'_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_{n+1,1} & \ldots & a_{n+1,n+1} & v'_{n+1} \\ v'_1 \; v'_2 & \ldots & v'_{n+1} & 0 \end{vmatrix}, \quad M = -\begin{vmatrix} a_{11} & \ldots & a_{1,n+1} & v'_1 \\ \cdot & \cdot & \cdot & v'_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_{n+1,1} & \ldots & a_{n+1,n+1} & v'_{n+1} \\ 0 \; 0 & & -1 & 0 \end{vmatrix} \\ U &= -\begin{vmatrix} a_{11} & \ldots & a_{1n} & v'_1 \\ \cdot & \cdot & \cdot & v'_2 \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \ldots & a_{nn} & v'_n \\ v'_1 & \ldots & v'_n & 0 \end{vmatrix} \end{aligned}\right\} \quad (25)$$

welche der Bedingung $C\Phi = M^2 + DU$ genügen.

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[4]) S. 166 steht im letzten Glied von (32) $\sqrt{\frac{R}{S}}$ statt $\sqrt{\frac{S}{R}}$.

Als absolute Invarianten kann man wählen

$$k_n = D^n\, C^{-(n+1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

und $l_n = \dfrac{C\,\Phi}{M^2}$ oder

$$l_n' = \frac{l_n}{l_n - 1} = \frac{C\,\Phi}{D\,U} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

k_n ist dabei wieder eine Invariante nur von f; l_n bzw. l_n' sind Invarianten für die allgemeine affine Gruppe. k_n hängt für ein Hyperellipsoid in einfacher Weise mit dem Volumen F_n zusammen. Wie man leicht nachgeht hat man [5])

$$\left.\begin{aligned} k_{2m} &= (m\,!)^2\, \pi^{-2m}\, F_{2m}^2 \\ k_{2m+1} &= -1^2 \,.\, 3^2 \,.\, 5^2 \ldots (2m+1)^2 \,.\, 4^{-m-1}\, \pi^{-2m}\, F_{2m+1}^2 \end{aligned}\right\} \quad . \quad (28)$$

Die Invariante l' hat eine geometrische Bedeutung, welche mit der für den Fall $n=2$ gefundenen analog ist. Ist $f=0$ ein Hyperellipsoid, aus welchem $(v'x)=0$ die quadratische Varietät $V((n-2)$-ter Dimension) und der mit $(v'x)=0$ paralleler Raum durch den Mittelpunkt die quadratische Varietät V_1 ausschneidet, so sind V und V_1 homothetische Figuren; die Homothetie ist bekanntlich ein für die allgemeine affine Gruppe invarianter Begriff. Die Homothetie-Zentren liegen auf demjenigen Durchmesser des Ellipsoids, welcher mit $(v'x)=0$ konjugiert ist. Sind g bzw. $-g$ die Faktoren derjenigen Homothetien, welche V_1 in V überführen, so ist

$$l' = g^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Der Beweis dieser Behauptung kann man, wenn man auf die Benutzung eines bestimmten Koordinatensystems verzichten will, in einer Weise führen, welche mit der von uns für $n=2$ gegeben wesentlich übereinstimmt. Wählt man ein geeignetes Koordinatensystem, so hat man für f und $(v'x)$ bzw. folgende Formen:

$$x_1^2 + x_2^2 + \ldots x_n^2 - R^2\, x_{n+1}^2 \quad \text{und} \quad x_n - p\, x_{n+1}$$

woraus hervorgeht

$$D = -R^2,\ C = 1,\ \Phi = p^2 - R^2,\ M = p,\ U = 1,\ k = (-1)^n\, R^{2n},\ l' = \frac{R^2 - p^2}{R^2}$$

[5]) Für $n=3$ BLASCHKE, l.c. S. 112.

Die Volumen F'_n und F''_n der beiden Teile, worin das Ellipsoid von dem Raum $(v'x) = 0$ zerteilt wird, können mittels bekannter Formeln für das Volumen eines Hyperkugelsegmentes [6]) niedergeschrieben werden.

Für $n = 3$ hat man im Besonderen:

$$\left.\begin{array}{l} F'_3 : F''_3 = \{2 - (2 + l')\sqrt{1 - l'}\} : \{2 + (2 + l')\sqrt{1 - l'}\} \\ F'_3 + F''_3 = \frac{4}{3}\pi\sqrt{-k} \end{array}\right\} . \qquad (30)$$

welche Resultate mit den WEITZENBÖCKschen (S. 177, (25) und (26)) nach einer Substitution übereinstimmen.

[6]) SCHOUTE, Mehrdimensionale Geometrie II, S. 290 (1905).

Chemistry. — *On geometric isomerism in Luteo-Cobaltic-salts.* By I. LIFSCHITZ and K. M. DIJKEMA. (Communicated by Prof. F. M. JAEGER.)

(Communicated at the meeting of June 29, 1940.)

§ 1. According to the classical co-ordination theory, it might be expected, that complex compounds of trivalent metals, such as, for instance: **I.** $[Met.^{III}A_3]$; **II.** $[Met.A\text{-}B\text{-}X_2]$; **III.** $[Met.A_2X_2]$, etc., in which A and B are co-ordinatively-bivalent asymmetrical substituents, X being a co-ordinatively-monovalent substituent, — wil occur in geometrically isomeric forms. *Luteo*-salts I, for instance, of the ion: $[Met.^{III}(pn)_3]^{\cdots}$ and *triacidotriammino*-salts *I*, for instance: $[Met.^{III}(glyc)_3]$, would be obtainable in *two*, some *diacidotetrammino*-salts *III*, for instance, of the ion: *flavo*-$[Met.^{III}(pn)_2\ (NO_2)_2]^{\cdot}$, even in *three* isomeric forms. Experimentally such isomerism has been observed only in cobaltic compounds and even there only in a few cases. In the first place geometrically isomeric forms were found in complex cobaltic compounds with 3 mol. α-amino-acid [1]). A. WERNER could prepare the isomeric forms of *flavo*-$[Co\,(en)\,(pn)\,(NO_2)]_2X$. [2]). But even *flavo*-$[Co\,(pn)_2(NO_2)_2]X$, of which three isomerides might theoretically be expected, appears only to occur in *one* single form [3]).

The reasons why the geometric isomerism predicted by WERNER is so rarely observed, will not be discussed here. In the first place it must experimentally be investigated, whether a greater number of geometrically isomeric salts *can* be obtained.

Apart from this, however, the preparation of such isomerides in an optically-active form would be of interest with respect to the study of rotatory power and rotatory dispersion in general.

Since geometrically isomeric substances differ almost exclusively with respect to the *degree of symmetry* of their molecules, an investigation of

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[1]) C.f. I. LIFSCHITZ, Proc. Kon. Akad. v. Wetensch., Amsterdam **27**, 721 (1924); **39**, 1192 (1936); **42**, 173 (1939).

[2]) A. WERNER, Helv. Acta I, 5 (1918).

[3]) The statements of MIKLOSICH, who thought that he had isolated the 3 forms expected, are incorrect. MIKLOSICH in his experiments started from *commercial* propylenediamine, which nearly always contains ethylenediamine, as was proved by the experiments of one of us (LI). As a matter of fact., H. E. WATTS (Diss. Zürich 1912) and H. HÜRLIMANN (Diss. Zürich 1918) failed to obtain such isomerides in using *pure* propylenediamine.

their rotatory power may yield important data concerning the influence of differences in symmetry on rotatory power [1]).

§ 2. In an investigation of the complex-formation of *phenylated ethylenediamines*, it was found that the *luteo*-cobaltic-salts of *monophenyl-ethylenediamine* can rather easily be obtained in geometrically isomeric forms. This is the first case of geometric isomerism in the *luteo*-series having been demonstrated and of the polarimetric investigation of such isomers having become possible.

Phenyl-ethylenediamine: $C_6H_5CH(NH_2)—CH_2(NH_2)$ (*phenen*) was obtained in the racemic and in the optically-active forms by catalytic hydrogenation of *a-acetaminophenyl-acetic-acid-nitril*:

$$C_6H_5CH\,.\,(NHCOCH_3)\,.\,CN$$

according to H. REIHLEN's method [2]). In preparing the *luteo*-salts, the following prescription may be followed as well in the case of the racemic as of the optically-active bases.

20,4 g. phenyl-ethylenediamine and 4,2 cc HCl (s.g. 1.19) are added to a solution of 11.9 g $CoCl_2 . 6H_2O$ in 350—400 cc methylalcohol and a current of air is blown through it by means of a gasdispersion-tube. Very soon a yellow salt crystallizes, whilst the solution becomes dark brown. After ca. 20 hours the solid salt (L_1) is sucked off, boiled three times with methyl-alcohol (each time with ca. 100 cc.) and, after complete cooling, sucked off.

The salt thus obtained is then fractionated by crystallisation from water, whilst some cc. of HCl (1,4) are added to the filtered and slightly cooled solution [3]).

Besides some luteo-salt L_1 the deep-brown methyl-alcoholic mother liquor of L_1 contains a deep-brown, occasionally somewhat greyish-coloured salt, which, after evaporation, is left as a syrupy substance, which on cooling solidifies into a crystalline mass. It probably consists of a polynuclear salt, which, however, even by re-crystallization from water [4]), could not be obtained in a pure state.

When this salt is dissolved in hot water, to which some cc hydrochloric acid (1 : 4) are added, and the solution is filtered and cooled, the colour changes to a yellowish-brown and a yellowish-brown salt (L_2) crystallizes from it; the latter can be purified by fractionated crystallisation from very dilute *HCl* in the way previously described.

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[1]) I. LIFSCHITZ, Rec. Trav. Chim. Pays-Bas **58**, 785 (1939); more extensive literature is given there.

[2]) H. REIHLEN, Ann. **493**, 20 (1932); **494**, 157 (1932).

[3]) L_1 is considerably less soluble in dilute HCl than in pure water; moreover, in this way a brown colouring matter is removed (see following pages).

[4]) After re-crystallization from water, one finds, for instance, $Cl = 14{,}80\,\%$; $14{,}75\,\%$; N (micro) $= 13{,}01\,\%$; $13{,}06\,\%$; $H_2O = 6{,}9\,\%$ so that $N : Cl = 2{,}24 : 1$.

This salt appears to be the *luteo*-salt L_2, which is geometrically isomeric with L_1. The dry solid salts differ but little in colour, — L_2 having a little darker yellow hue. Analysis yielded the following values:

	Co:	Cl:	N:	H_2O:
L_1	9.72 %; 9.78 %	17.51 %; 17.56 %	13.77 %; 13.67 %	5.0 %
L_2	9.39 %; 9.36 %	16.89 %; 16.83 %	13.44 %; 13.31 %	8.6 %; 9.2 %
calculated for $[Co(phenen)_3]Cl_3 \, . \, 3\,H_2O$:	9.39	16.98	13.39	8.6 %
calculated for $[Co(phenen)_3Cl_3] \, . \, 2\,H_2O$:	9.67	17.48	13.78	5.7 %.

Thus L_1 appears to be a *di*hydrate, L_2 a *tri*hydrate: A determination of the molecular weight of the dried salts in phenol, yielded the following values:

	gr. salt	gr. phenol	Δt:	M (found):	
L_1	0.293	17.25	0°.210	591	
L_1	0.290	16.23	0°.222	596	M (calculated) $= 573$
L_2	0.391	16.07	0°.290	607	

Both salts, therefore, are *mononuclear* ones and have *the same* molecular weight. The difference of colour of the solid salts is also noticeable in aqueous solutions: solutions of the same concentration by means of a PULFRICH-photometer proved to show an absorption of different extent. In methyl-alcohol L_1 dissolves with difficulty, L_2 rather easily.

The saturated aqueous solution of L_1 at 25° contains 1.16 gr. that of L_2 2.47 gr. of salt in 100 cc of water. In hot, boiling water both salts readily dissolve, L_2 melting under water to a brown liquid mass, whereas L_1 remains solid, until complete solution has occurred.

On heating in a drying room, the two salts become green, L_2 at ca. 140°, L_1 at 150°, under partial conversion into *praseo*-salts.

The data concerning the rotation-curves of the two isomerides are to be found in Fig. 1, (curves *I* and *II*). It is noteworthy that, — in contradistinction to the case of *l*-stilbenediamine, — *l*-phenylethylenediamine yields *levo*-rotating *luteo*- and *praseo*-salts. The rotation-curves of the two isomerides show the same qualitative course, but they strongly differ in regard to the *absolute* magnitude of their rotations. Both isomerides possess a considerably higher maximal rotatory power than $[Co(l\text{-}stien)_3]Cl$. As the curve shows, the *l*-base gives a *positive* contribution to the rotation [1]); the partial racemic complexes: $[Co(rac.\ phenen)_3]X_3$ therefore, ought to possess a much higher rotatory power. The polarimetric investigation of the geometrically isomeric *luteo*-salts fully confirms the rules [2]) previously stated by one of the authors (LI) with other geometrically isomeric series:

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[1]) In the red part of the spectrum, where the luteo-complexes practically have *no* rotation, the said contribution is very evident, see curve II.

[2]) I. LIFSCHITZ, l.c. Proc. Ned. Akad. v. Wetensch. Amsterdam, l.c.

1. Geometrically isomeric substances show a rotation which is *qualitatively quite analogous*, but *quantitatively very different*.

2. A *lower* degree of symmetry of the molecule causes a *higher* rotation: $[Co(l\text{-}stien)_3]X_3$ rotates much less than $[Co(l\text{-}phenen)_3]X_3$ and practically as much as $[Co(en)_3]\ X_3$ [1]).

3. From this, the working hypothesis obtains fresh support [2]), that of two geometric isomerides the *less symmetrical* one shows the *higher* rotation.

§ 3. Besides *luteo*-salts, we also prepared the *praseo*-compounds; theoretically these might also occur in *two* isomeric forms, namely in a *cis*- and a *trans*-form. We, indeed, got the impression that *two* praseo-chlorides $[Co(phenen)_2Cl_2]Cl$ are formed; but they were only little stable and, therefore, they could up till now not be isolated in the pure state with complete certainty.

For their preparation the following prescription is recommended:

7.8 gr. of $CoCl_2 . 6\,H_2O$ are dissolved in 300 cc alcohol, 8.1 gr. base and then 10 cc HCl (s.g. 1.19) are added and the mixture then is oxydized with a current of air during about 20 hours. A green salt soon begins to crystallize, the solution assuming a blackish-green colour.

The crystallized *praseo*-salt (P_1) is sucked off and the mother liquor strongly concentrated; on cooling, a considerably darker salt (P_2) then begins to crystallize. Both salts, after recrystallizing them from alcoholic hydrochloric acid and drying at 130°, are, however, still impure. A pure *praseo-nitrate* could finally be obtained by mixing a solution of P_1 in methyl-alcohol with an aqueous solution of KNO_3. This salt, which dissolves in water with great difficulty and slightly better in alcohol, gave the following values:

$[Co(phenen)_2Cl_2]NO_3$ Calc.: $Cl = 15{,}31\ \%$; $N = 15{,}09\ \%$; found: $Cl = 15{,}44\ \%$; $15{,}54\ \%$; $15{,}26\ \%$; $N = 14{,}83\ \%$; $14{,}88\ \%$. The values for nitrogen, which are a little too low, are caused by the presence of *Cl* in the molecule [3]). The data concerning the rotation-curve of the *praseo*-salt, which shows a course perfectly analogous to that of the curves of other *prasea*-compounds [4]), is given in Fig. 1 by curve *III*. By treating the *praseo-chlorides* with potassium- or silver-oxalate, an *oxalo-salt*: $[Co(phenen)_2C_2O_4)]Cl$ finally also could be obtained. The salt crystallizes from water in red little prisms, containing 2 molecules of water: Calc.: $N = 11{,}42\ \%$; $Cl = 7{,}2\ \%$; found: $N = 11{,}33\ \%$ and $11{,}40\ \%$; *Cl* 7,2 and 7,0 %. Curve *IV* in Fig. 1 graphically represents the rotation of this salt. It can be remarked, that the *oxalo*-salt:

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1) I. LIFSCHITZ and J. G. BOS; this paper will soon appear in Rec. trav. chim. d. Pays-Bas, **59**, (1940).

2) I. LIFSCHITZ, Rec. trav. chim. d. Pays-Bas, **58**, 785 (1939).

3) I. LIFSCHITZ and J. G. BOS, Rec. trav. chim. **58**, 795 (1939).

4) F. M. JAEGER and H. BLUMENDAL, Z. anorg. allg. chem. **175**, 191 (1928).

$[Co(phenen)_2C_2O_4]Cl$ may occur in *three* geometrically isomeric forms. An isomeric salt, however, was in our experiments only found once, in a

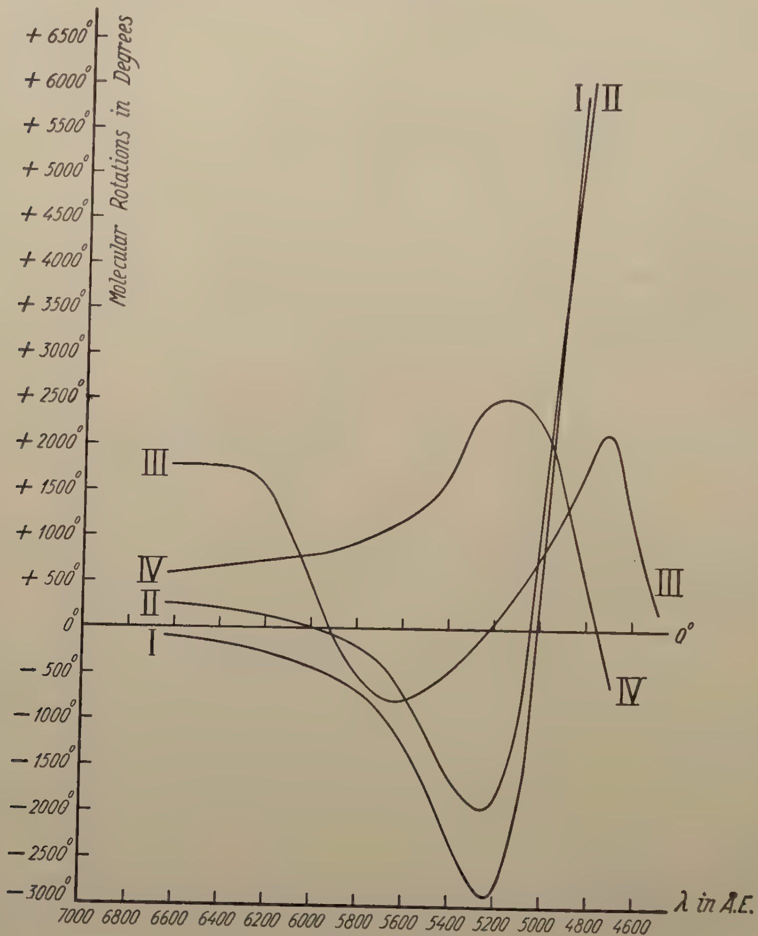

Fig. 1. Rotatory Dispersion of:

I. Luteo-salt L_1 in water, ($c = 0{,}2252$ gr. in 100 cc).
II. Luteo-salt L_2 in water, ($c = 0{,}2244$ gr. in 100 cc).
III. Praseo-salt: $[Co(phenen)_2Cl_2]\,NO_3$ in alcohol + 5 % conc. HCl ($c = 0{,}1300$ gr. in 100 cc).
IV. Oxalo-salt: $[Co(phenen)_2C_2O_4]Cl$ in water ($c = 0{,}1180$ gr. in 100 cc).

minimal quantity. Because of the lack of material, for the present we have abstained from more extensive investigations. The determination of N of this salt yielded: 11,40 and 11,50 % N; calculated for the compound: $[Co(phenen)_2C_2O_4]Cl \cdot 2\,H_2O$: 11,42 %.

Groningen, Laboratory for Inorganic and Physical Chemistry of the University.

Plantkunde. — *Snelle bloei van Iris Wedgwood.* (with summary). (Mededeeling N⁰. 64 van het Laboratorium voor Plantenphysiologisch Onderzoek te Wageningen). Door ANNIE M. HARTSEMA en IDA LUYTEN. (Communicated by Prof. A. H. BLAAUW).

(Communicated at the meeting of June 29, 1940.)

Nadat wij in het voorjaar van 1934 voor het eerst bol-irissen hadden vervroegd, n.l. de uit Marokko in Zuid-Frankrijk geïmporteerde *Iris tingitana* (Med. N⁰. 40, 1934), werden in den zomer van 1934 ook proeven ingezet over het vervroegen van den bloei van enkele Hollandsche Irissen, o.a. de variëteiten Imperator en Wedgwood. Over de resultaten van onze proeven met Iris Imperator verschenen reeds 2 publicaties, Med. N⁰. 48, 1936 en Med. N⁰. 57, 1939. Thans zullen wij de proeven met Iris Wedgwood beschrijven.

Zooals in Med. N⁰. 62, 1939, werd vermeld, ontstond de variëteit Wedgwood door kruisingen met of van de Marokkaansche *Iris tingitana*. In 1934/35 werden voor 't eerst proeven genomen om den bloei van Wedgwood te vervroegen o.a. met de behandeling die wij voor *Iris tingitana* hebben beschreven, n.l. 3 weken 28° gevolgd door 9°. In geen van deze proeven werd echter bloei bereikt.

In 1935/36 werd daarom met Wedgwood een groot aantal proeven genomen om te kunnen vaststellen, in hoeverre andere temperatuurbehandelingen betere resultaten zouden geven. De bollen van Wedgwood van de maat 10—11 cm (omtrek) waren op 7 Augustus 1935 ontvangen. Voor iedere proef werden 2 × 10 stuks gebruikt, bestemd voor 2 kistjes. De bollen werden uitgezocht op een gewicht van 22—33 gram, 10 stuks wogen 260 gram. Ten deele werden de bollen direct geplant in 5°, 7°, 9° of 13° C.; de andere groepen van 20 stuks werden eerst gedurende 1, 3 of 5 weken bij een hoogere temperatuur gelegd, en daarna eveneens bij 5°, 7°, 9° of 13° C. geplant. Als hoogere vóórtemperatuur was, evenals bij de proeven met Imperator 23°, 28° en 31° C. gekozen. De zin van deze warme *vóórtemperatuur* is in Med. 48 p. 605 uiteengezet. De lage temperaturen (5°, 7°, 9° en 13°) waarbij geplant werd, noemden wij *prepareer-temperaturen*. Pas nadat de planten een spruit van een zekere lengte ontwikkeld hebben, komen ze in de *trektemperatuur*, n.l. in een matig-warme kas van 15° C., waarin de strekking van de spruiten verder voortgezet wordt en tenslotte de bloemen zich ontplooien.

Nu bleek achterna, dat niet in alle bollen een bloem aangelegd was en dat

't al of niet aanleggen van de bloem samenhing met de toegepaste temperatuur-behandeling. In tabel 1 wordt aangegeven hoeveel bloemen per groep van 20 bollen gevormd waren.

TABEL 1. 1935/36.

vóórtemperatuur:		—	23°			28°			31°
gedurende		—	1 wk.	3 wk.	5 wk.	1 wk.	3 wk.	5 wk.	1 wk.
prepareer-	5°	0	5	15	19	3	16	19	5
temperatuur	7°	0	8	17	18	11	18	19	9
	9°	1	15	20	20	13	19	20	14
	13°	17	18	19	20	20	20	20	20

Uit deze tabel blijkt wel dat voor Wedgwood een voorbehandeling met een hoogere temperatuur noodzakelijk is voor het slagen van den bloemaanleg, daar zonder deze voorbehandeling (zie 1e kolom) bij geen der gebruikte prepareer-temperaturen alle 20 bloemen aangelegd werden. Bovendien blijkt, dat de inwerking van deze hoogere temperatuur gedurende 1 week in het algemeen te kort is, omdat alleen in 13° na 1 week 28° of 1 week 31° alle bloemen gevormd werden. Naarmate de voorbehandeling bij 23° of 28° langer geduurd heeft, zien wij dat meer bloemen aangelegd worden; 31° werd in dit jaar niet langer dan 1 week toegepast. Maar zelfs na 5 weken 23° of 28° werden in 5° en 7° nog niet alle bloemen aangelegd. Dit was uitsluitend het geval in 9° en 13°.

Het was niet te verwachten, dat de aangelegde bloemen in deze eerste proeven ook alle tot bloei zouden komen. Evenals bij de eerste proeven met Imperator, werden de kistjes bij het bereiken van een neuslengte van tenminste 6 cm (dwz. dat alle of bijna alle spruiten 6 cm of meer buiten den bol steken) uit de prepareertemperatuur overgebracht naar een kas van 15°. In tabel 2 geven wij alleen de resultaten van die proeven, waarbij alle bloemen aangelegd waren. Ook in enkele andere proeven ontplooiden zich bloemen, maar het heeft geen zin, deze hier te vermelden. Zouden wij alle

TABEL 2. 1935/36.

behandeling	6 cm bereikt	aantal dagen na planten	1e bloem open	aantal dagen na 6 cm	aantal dagen na planten	totaal aantal bloemen
3 w. 23° — 9°	31 Oct.	65	7 Jan.	68	133	3
5 w. 23° — 9°	6 Nov.	57	—	—	—	0
5 w. 28° — 9°	11 Nov.	62	14 Jan.	64	126	9
5 w. 23° —13°	21 Oct.	41	10 Jan.	81	122	14
1 w. 28° —13°	1 Oct.	49	18 Dec.	78	127	4
3 w. 28° —13°	9 Oct.	43	26 Dec.	78	121	4
5 w. 28° —13°	21 Oct.	41	3 Jan.	74	115	8
1 w. 31° —13°	30 Sept.	49	—	—	—	0

toegepaste behandelingen weergeven, dan zou daaruit blijken dat er evenals bij Imperator (vergelijk tabel 1 en 2 van Med. 48) een verband bestaat tusschen de gebruikte prepareer-temperaturen en de tijden die noodig zijn om na het planten 6 cm neuslengte te bereiken en evenzeer tusschen deze temperaturen en de tijden, die na het bereiken van 6 cm neuslengte tot het begin van den bloei verloopen.

Ook uit bovenstaande tabel 2 blijkt, dat na planten in 13° eerder 6 cm bereikt wordt dan na planten in 9°, terwijl omgekeerd de tijd tusschen het bereiken van 6 cm en het begin van den bloei bij 9° korter is dan bij 13°. In tegenstelling met onze ervaringen bij Imperator is bij Wedgwood de som van de genoemde tijden na planten bij 9° *niet korter* dan na planten in 13°. Voor snellen bloei van Wedgwood is 13° dan ook te verkiezen boven 9°. Overigens zijn de bloei-resultaten van de hier genoemde proeven nog ongunstig. Slechts in één geval werd een behoorlijk bloei-percentage, nl. 70 % bereikt, en wel na een voorbehandeling van 5 weken 23°, gevolgd door planten in 13°. Opvallend is dat na 5 weken 23° bij planten in 9° geen enkele bloem opengegaan is. In het algemeen blijkt uit tabel 2 wel, dat ook voor den bloei een langer-durende voorbehandeling te verkiezen is boven korter-durende. Men verliest daardoor echter zooveel tijd, dat de bloei van Wedgwood niet veel eerder begint dan die van Imperator (vergelijk Med. 48 blz. 608, waar 80 % bloei bereikt kon worden na een behandeling met 1 week 31°, gevolgd door planten bij 9°; begin van den bloei reeds op 14 Januari.)

Daarom moest gezocht worden naar methoden om beteren en zoo mogelijk ook vroegeren bloei te verkrijgen. In de eerste plaats werd op aanwijzing van kweekers niet direct na de warme voorbehandeling in de lage prepareertemperatuur geplant, doch pas 6 weken later. Gedurende deze 6 weken werden de bollen droog bewaard bij de prepareertemperatuur. Naar wij vernamen had men daarmee in de practijk goede resultaten bereikt. Ook GRIFFITHS (1936) vermeldt, dat men vroegen bloei van Wedgwood kan verkrijgen door de bollen na een voorbehandeling met 80° F. (26½° C.) gedurende de maand Juli (de bollen werden reeds begin Juli gerooid) minstens 4 weken bij 50° F. (9° C.) te bewaren.

In de tweede plaats werden, uit dezelfde overwegingen als wij in Med. 57 reeds vermeldden, de kistjes niet direct bij het bereiken van tenminste 6 cm neuslengte overgebracht naar de kas van 15°. Het tijdstip van dit overbrengen bij 6 cm was immers willekeurig gekozen en uit andere onderzoekingen bij Iris Imperator was gebleken, dat de bloemen pas vrij laat en meestal 't snelst bij lage temperatuur (7 à 9° C.) gevormd worden. Het leek daarom gunstiger de bloemen eerst grootendeels te laten afmaken in de lage prepareertemperatuur en het tijdstip van overbrengen niet te laten bepalen door de lengte van de spruit, doch door het stadium van de bloem.

In 1936/37 werd als prepareer-temperatuur uitsluitend 9° C. gekozen, terwijl de bollen voorbehandeld werden met 31° C. gedurende 1 week.

overeenstemmend met de voor snellen bloei van Imperator vastgestelde behandeling. Verder werden dus de bollen 6 weken droog bewaard bij 9° en daarna geplant bij dezelfde temperatuur. Na het bereiken van minstens 6 cm neuslengte werden de kistjes nog gedurende 2 of 4 weken bij 9° gelaten en wel in een koude kas, waar de spruiten dus in het licht stonden. Na deze 2 of 4 weken werd van iedere groep een kistje gefixeerd en werden 2 kistjes naar kas 15° C. overgebracht. Zooals te verwachten was, waren ook ditmaal niet in alle bollen bloemen aangelegd, van 30 gefixeerde bollen hadden 5 geen bloem gevormd. Uit de fixaties bleek dat bij het bereiken van 6 cm de bloemen reeds in stadium IV tot V verkeerden, 2 weken later was stadium V tot VI bereikt, en 4 weken later waren alle bloemen geheel klaar. Daar bij geen dezer proeven 50 % bloei werd bereikt, zullen wij de resultaten hier niet vermelden.

In 1937/38 werd overgegaan tot langere voorbehandelingen en wel 3 weken 31° of 5 weken 28°. Als prepareer-temperatuur werd behalve 9° ook 11° gebruikt; ook nu bleven de bollen na de voorbehandeling nog 6 weken droog bewaard bij de prepareer-temperatuur voordat ze geplant werden. De bollen waren ontvangen op 13 Augustus 1937 en ze werden uitgezocht tusschen 22 en 26 gram (gewicht per 10 stuks 233 gram). Voor iedere proef werden ditmaal 3 kistjes van 10 stuks gebruikt. Voorzichtigheidshalve werd ook nu niet direct bij het bereiken van 6 cm neuslengte overgebracht naar een kas van 15°, doch eerst nog 3 weken naar een kas van 9° resp. 11° C. De resultaten vindt men in tabel 3 vermeld.

TABEL 3. 1937/38.

voorbehandeling	ongeplant bewaard	6 cm bereikt (in 9° of 11°)	over n. kas 15°	1e bloem open	totaal aantal bloemen
3 w. 31°	6 w. 9°	15 Nov.	6 Dec.	21 Jan.	21 : 30
3 w. 31°	6 w. 11°	8 Nov.	29 Nov.	11 Jan.	17 : 30
5 w. 28°	6 w. 9°	25 Nov.	16 Dec.	30 Jan.	17 : 30
5 w. 28°	6 w. 11°	18 Nov.	9 Dec.	22 Jan.	16 : 30

Ook in dit geval begon de bloei vrij laat; het beste resultaat, 70 % bloei, werd bereikt bij de eerste groep: 3 w. 31 ° en vervolgens 9°. De 2e groep: 3 w. 31° gevolgd door 11° begon echter 10 dagen eerder te bloeien. Bij deze proeven bleken alle bloemen te zijn aangelegd. Dit is weer een bewijs hoe noodzakelijk de voorwarmte is. Verder werd ook hier van iedere groep een kistje gefixeerd op het oogenblik van overbrengen naar kas 15°; vastgesteld kon worden dat de bloemen toen reeds geheel gereed waren. Wij komen hierop nog nader terug.

In 1938 werden al onze Iris-proeven in plaats van in het tot nu toe gebruikte zuivere duinzand uitgeplant in voedzamen grond, samengesteld uit $^3/_5$ bladgrond, $^1/_5$ verteerde koemest en $^1/_5$ scherp zand, vermengd met

2 kg Thomasslakkenmeel per m^3; aan dit grondmengsel werd toegevoegd ¼ volume fijn gemalen baksteen (gravel) om het luchtgehalte te verhoogen. Aanleiding daartoe waren de resultaten van onze proeven met Iris Imperator in 1937/38 (zie Med. N⁰. 57) waarbij geconstateerd werd, dat extra voeding vóór of gedurende het trekseizoen, een zeer gunstigen invloed op het bloeipercentage had. Door een technische vergissing bij deze serie was het resultaat van den voedzamen grond in dit jaar te slecht om te beoordeelen.

Daarom moesten deze proeven in 1939/40 herhaald worden, waarbij deze fout vermeden werd. De bollen werden in den zomer van 1939 pas op 21 Augustus gerooid en konden niet eerder dan op 23 Augustus bij 31° resp. 23° geplaatst worden. Ditmaal werden als voorbehandelingen gekozen 3 weken 31° en 5 weken 23°. Ook nu werden de bollen na de warme voorbehandeling nog gedurende 6 weken droog bewaard en wel bij 9°, 11° en 13°. Ter contrôle werd één groep direct bij 9° geplant, terwijl een andere contrôle-groep niet in voedzamen grond vermengd met gravel, doch in zuiver zand, eveneens vermengd met gravel, geplant werd. De bollen werden ditmaal uitgezocht tusschen 25 en 30 gram, gewicht per 10 stuks 259 gram. Voor iedere proef werden 2 × 10 stuks gebruikt. Wij zullen nu eerst het resultaat van de contrôleproeven vermelden in vergelijking met een overeenkomstige proef van een der later te bespreken series. Uit tabel 4 blijkt, dat de bloei van de 6 weken droog bewaarde bollen enkele dagen later begon dan die van de direct-geplante (beide na dezelfde voorbehandeling). Het gebruik van voedzamen grond in plaats van zuiver duinzand gaf enkele dagen vervroeging van den bloei maar bovendien een veel krachtiger gewas. De stengels waren in de zandproeven niet alleen dunner maar ook merkbaar korter, de gemiddelde lengte was 65 cm in plaats van 70 cm.

TABEL 4. 1939/40.

behandeling	grondsoort	6 cm bereikt	1e bloem open	totaal aantal bloeiend	aantal dagen 1e tot laatste bloem
3 w. 31° — 9°	voedz. grond	13 Nov.	24 Jan.	20	9
3 w. 31° + 6 w. 9°	voedz. grond	20 Nov.	26 Jan.	20	8
3 w. 31° + 6 w. 9°	zand	21 Nov.	29 Jan.	20	7

Bij deze proeven kwamen steeds alle bloemen open, wat waarschijnlijk toegeschreven moet worden aan het zeer gunstige jaar, zooals ook uit de andere tabellen zal blijken.

Alle andere proeven werden dus geplant in voedzamen grond vermengd met gravel. De eerste serie werd bij het bereiken van 6 cm neuslengte direct overgebracht naar kas 15°. De 2e en 3e serie brachten wij na 6 cm nog gedurende 10 dagen resp. 3 weken in een kas van dezelfde temperatuur

als waarbij geplant was; de 4e serie werd evenals de 3e gedurende 3 weken in een koelere kas gehouden, vervolgens in een kas van 15° geplaatst en tenslotte overgebracht naar een kas van 17° op het moment dat bij de meeste planten het 5e loofblad zichtbaar was. Wij zullen deze vier series afzonderlijk in tabellen weergeven en bespreken (tabel 5—8).

TABEL 5. 1e serie 1939/40.

voorbehandeling	ongeplant bewaard	6 cm bereikt (in 9°, 11° of 13°)	aantal dagen na 't planten	1e bloem open	aantal dagen na 6 cm	totaal aantal bloeiend	aantal dagen 1e tot laatste bloem
3 w. 31°	6 w. 9°	20 Nov.	25	16 Jan.	57	20	7
3 w. 31°	6 w. 11°	13 Nov.	18	11 Jan.	59	19 (1)	10
3 w. 31°	6 w. 13°	11 Nov.	16	10 Jan.	60	18 (2)	6
5 w. 23°	6 w. 9°	30 Nov.	22	5 Febr.	67	19	6
5 w. 23°	6 w. 11°	25 Nov.	17	29 Jan.	65	20	7
5 w. 23°	6 w. 13°	23 Nov.	15	30 Jan.	68	19 (1)	7

Zooals te verwachten was, werd 6 cm neuslengte eerder bereikt na planten in 13° dan na planten in 9° of 11°. Toch is het verschil tusschen 9° en 13° niet zoo groot als bijv. in tabel 2. Vergelijken wij de met 5 weken 23° voorbehandelde groepen, dan blijkt uit tabel 2, dat in 13° 6 cm neuslengte 16 dagen eerder bereikt werd dan in 9°, terwijl in tabel 5 dit verschil slechts 7 dagen bedraagt. Nog duidelijker was de voorsprong in tabel 2 na 5 weken 28°: in 13° 21 dagen eerder dan in 9°! Bij het in bloei komen was deze voorsprong verminderd tot 10 dagen. In tabel 5 zien we de voorsprong zelfs geheel verdwijnen, maar toch komen de in 13° en 11° geplante bollen nog eerder in bloei dan de in 9° geplante, in tegenstelling met onze ervaringen bij I m p e r a t o r, waar 9° of 7° den snelsten bloei geven.

In tabel 6 worden de proeven van de 2e serie samengevat. Het aantal dagen dat na het planten verloopt totdat 6 cm neuslengte bereikt wordt, is in al deze proeven hetzelfde en wordt dus niet herhaald.

TABEL 6. 2e serie 1939/40.

voorbehandeling	ongeplant bewaard	over n. kas 15° (10 dg. na 6 cm)	1e bloem open	aantal dagen na 6 cm	totaal aantal bloeiend	aantal dagen 1e tot laatste bloem
3 w. 31°	6 w. 9°	30 Nov.	22 Jan.	63	19	7
3 w. 31°	6 w. 11°	23 Nov.	15 Jan.	63	19	6
3 w. 31°	6 w. 13°	21 Nov.	13 Jan.	63	18 (1)	6
5 w. 23°	6 w. 9°	10 Dec.	5 Febr.	67	19	6
5 w. 23°	6 w. 11°	5 Dec.	30 Jan.	66	19 (1)	6
5 w. 23°	6 w. 13°	4 Dec.	29 Jan.	67	19 (1)	11

Vergelijken we de data van het opengaan der 1e bloemen in deze tabel met die van het bereiken van 6 cm, dan blijkt dat er in beide groepen vrijwel een constant aantal dagen tusschen ligt. Dat de bloei niet gelijktijdig begint is uitsluitend te wijten aan het verschil dat reeds bestond bij het bereiken van 6 cm. Bij de met 3 weken 31° voorbehandelde groepen begon de bloei iets later dan in tabel 5, bij de andere groepen ongeveer gelijktijdig.

In tabel 7 vindt men de proeven van de 3e serie. Vooral de met 3 weken 31° voorbehandelde proeven zijn hier weer enkele dagen in bloei verlaat door het later overbrengen naar kas 15°; van de met 5 weken 23° voorbehandelde is alleen de in 9° geplante merkbaar verlaat.

TABEL 7. 3e serie 1939/40.

voor-behan-deling	ongeplant bewaard	over n. kas 15° (3 wk. na 6 cm)	1e bloem open	aantal dagen na 6 cm	totaal aantal bloeiend	aantal dagen 1e tot laatste bloem
3 w. 31°	6 w. 9°	11 Dec.	26 Jan.	67	20	6
3 w. 31°	6 w. 11°	4 Dec.	18 Jan.	66	18	7
3 w. 31°	6 w. 13°	2 Dec.	14 Jan.	64	19	7
5 w. 23°	6 w. 9°	21 Dec.	10 Febr.	72	20	5
5 w. 23°	6 w. 11°	16 Dec.	1 Febr.	67	20	6
5 w. 23°	6 w. 13°	14 Dec.	1 Febr.	69	19 (1)	9

In alle drie deze proevenseries (tabel 5—7) *bloeien de met slechts 3 weken 31° voorbehandelde groepen eerder dan de met 5 weken 23° voorbehandelde.* Dit is begrijpelijk omdat de hooge voor-temperatuur uitsluitend een gunstige nawerking heeft op de bloemvorming en geen directe blad- of bloemvorming toelaat. *De vroegste bloei werd hier bereikt na de behandeling 3 weken 31° + 6 weken 13°, geplant in 13° en overgebracht naar kas 15° bij 6 cm neuslengte.* (Begin van den bloei 10 Januari, zie tabel 5). Bij deze behandeling maakt het niet veel verschil of men langer wacht met het overbrengen naar kas 15°, 3 weken later overbrengen geeft slechts 4 dagen verlating van den bloei. Bij de in 9° en 11° geplante groepen is het tijdstip van overbrengen iets meer van invloed op het begin van den bloei, hoe langer men wacht met overbrengen, hoe later de bloei begint.

Verder ziet men uit de tabellen 5—7, dat na al deze behandelingen 90—100 % bloei verkregen werd. In de kolommen, die het aantal bloemen aangeven, vindt men tusschen haakjes het aantal verdroogde bloemen vermeld. De andere niet-bloeiende bollen waren meer of minder ziek. Hoewel slechts weinig bloemen verdroogden, wat waarschijnlijk aan het zeer gunstige jaar te danken is, is het toch opvallend dat deze verdrogingen vooral bij de in 13° geplante groepen voorkomen. Verder volgt uit de tabellen 5—7, dat het voor het slagen van den bloei niet noodzakelijk is om langer dan tot 6 cm neuslengte te wachten met het overbrengen naar

kas 15°. Ook de voorbehandeling blijkt geen invloed te hebben op het bloeipercentage.

Het is nu de vraag of men den bloei nog meer vervroegen kan door in de plaats van een kas van 15° een kas van 17° te gebruiken. Deze proef werd in de 4e serie genomen, doch alleen met groepen die te vergelijken zijn met die van de 3e serie (tabel 7). Het overbrengen naar kas 17° geschiedde pas bij het zichtbaar worden van het 5e loofblad. Uit tabel 8 ziet men dat dit bij de behandeling 3 weken 31° + 6 weken 13° reeds 3 dagen na het overbrengen naar kas 15° mogelijk was, terwijl er bij de andere groepen wat langer gewacht moest worden.

TABEL 8. 4e serie 1939/40.

voorbehandeling	ongeplant bewaard	over n. kas 15°	over n. kas 17°	1e bloem open	aantal dagen na 6 cm	totaal aantal bloeiend	aantal dagen 1e tot laatste bloem
3 w. 31°	6 w. 9°	11 Dec.	25 Dec.	18 Jan.	59	20	9
3 w. 31°	6 w. 11°	4 Dec.	16 Dec.	12 Jan.	60	19	6
3 w. 31°	6 w. 13°	2 Dec.	5 Dec.	4 Jan.	54	16 (4)	7
5 w. 23°	6 w. 9°	21 Dec.	9 Jan.	3 Febr.	65	19 (1)	4
5 w. 23°	6 w. 11°	16 Dec.	27 Dec.	25 Jan.	61	20	3
5 w. 23°	6 w. 13°	14 Dec.	21 Dec.	21 Jan.	59	16 (1)	4

Hoewel het overbrengen naar kas 17° pas laat geschiedde, blijkt uit de vergelijking van tabel 7 en 8, dat hierdoor nog een duidelijke vervroeging kon worden bereikt. De snelstbloeiende groep van de vorige series (tabel 5): 3 weken 31° + 6 weken 13°, begon nu nog eenige dagen eerder te bloeien, n.l. reeds op 4 Januari, maar er verdroogden daarbij 4 van de 20 bloemen, zooals uit de tusschen haakjes geplaatste cijfers blijkt. Het is te verwachten dat het aantal verdroogde bloemen zou toenemen, indien men zou trachten den bloei nog meer te vervroegen, door bijv. reeds bij het bereiken van 6 cm naar kas 17° over te brengen. Wij zullen dit nader moeten onderzoeken.

Uit de laatste kolom van de tabellen 5—8 ziet men, dat in bijna alle proeven binnen een week na het opengaan van de 1e bloem, de beide kistjes geheel in bloei stonden. Opvallend snel was dit bij de 3 laatste groepen van tabel 8, waar in 3 tot 4 dagen alle bloemen opengingen.

De foto geeft een beeld van een bloeiende groep op 16 Januari 1940, na de behandeling: 3 weken 31° + 6 weken 11° (zie tabel 8, 2e groep). De bloei begon op 12 Januari, op de foto ziet men in het kistje links alle 10 bloemen reeds open; rechts zijn nog drie bloemen in knop, terwijl 1 zieke bol geen spruit ontwikkeld heeft. De stengels zijn zeer krachtig en stevig, de lengte gemeten van den bol tot de bloem bedroeg 65—72 cm (gemiddeld 70 cm). De kleur van de bloemen was mooi lichtblauw, had volstrekt niet geleden door het verblijf in de warmere kas van 17°. Een nadeel van de

Fig. 1. *Iris Wedgwood.* 3 wk. 31° + 6 wk. 11°; geplant bij 11°, bij 6 cm naar kas 11° gedurende 3 weken, daarna kas 15° en op 16 Dec. naar kas 17°; foto 16 Januari 1940.

bloemen van Wedgwood is dat deze, zelfs in een kas van 15°, reeds 4 à 5 dagen na het opengaan beginnen te verwelken, terwijl de bloemen van Imperator onder dezelfde omstandigheden tot 10 dagen goed blijven.

Hoewel onze bollen afkomstig waren van een op mozaïek-ziekte geselecteerde partij, merkten wij op de loof- en spatha-bladen nog wel kleine mozaïek-vlekken op. In enkele groepen kwamen ook planten voor, die duidelijker mozaïek-ziekte vertoonden; de stengel van zoo'n zieke plant bleef vaak korter, de bloem kwam langzamer open en verdroogde soms geheel of gedeeltelijk. Vlekken in de bloemen kwamen bij deze zieke planten nooit voor.

Van alle hier genoemde proeven werd zoowel bij het bereiken van 6 cm als 3 weken later telkens een kistje gefixeerd, om te kunnen nagaan, hoe het met den bloemaanleg stond. Ook nu weer bleek, dat de bloemaanleg van Wedgwood eerder begint dan die van Imperator (vergelijk ook Med. N°. 62), want bij het bereiken van 6 cm neuslengte, was de bloem reeds grootendeels gevormd. 3 weken later bleken alle bloemen geheel af te zijn, zoowel bij de groepen, die gedurende deze 3 weken in de kassen van lagere temperatuur waren blijven staan, als bij die welke reeds bij 6 cm naar kas 15° waren overgebracht. In het laatste geval was de strekking van het loof en de jonge bloemstengel duidelijk het sterkst. In tabel 9 geven wij alleen de stadia der hoofdbloemen bij het bereiken van 6 cm.

TABEL 9. Toestand bij het bereiken van 6 cm neuslengte.

voorbehandeling	ongeplant bewaard	fixatiedatum (6 cm neuslengte)	gem. aantal loofbl.	V−	V	V+	V—VI	VI−	VI
3 w. 31°	6 w. 9°	20 Nov.	5.0	—	1	—	8	1	—
3 w. 31°	6 w. 11°	13 Nov.	5.3	—	1	—	2	7	—
3 w. 31°	6 w. 13°	11 Nov.	5.9	—	2	—	5	1	1
5 w. 23°	6 w. 9°	30 Nov.	5.0	—	1	—	7	2	—
5 w. 23°	6 w. 11°	25 Nov.	5.0	—	1	—	1	7	—
5 w. 23°	6 w. 13°	23 Nov.	5.8	1	3	1	2	2	1

Uit tabel 9 blijkt dat reeds bij 6 cm neuslengte buiten den bol, alle bloemen in stadium V of verder zijn, d.w.z. dat alleen de vruchtbladen nog gevormd moeten worden.

Verder valt het op dat de in 11° gevormde bloemen het verst ontwikkeld zijn, terwijl de in 13° gevormde juist het minst ver zijn. Dit laatste is vooral daarom merkwaardig, omdat in 13° het eerst 6 cm wordt bereikt. Dit zou er op kunnen wijzen, dat 13° wel de snelste strekking geeft, maar niet de vlugste ontwikkeling van de bloemen. Deze conclusie is echter voorbarig, want uit de fixaties blijkt, dat er in 13° gemiddeld een loofblad meer gevormd is, dan in 9° of 11° (zie 3e kolom van tabel 9). Wij zullen nog nader moeten onderzoeken, of in 13° tengevolge van het grooter aantal

loofbladen de bloemvorming al dan niet later begint dan in 9° of 11°. Bij het begin der proeven op 24 Augustus, bedroeg het aantal loofbladen gemiddeld 4.1, zoodat dus in 9° en 11° gemiddeld slechts 1 loofblad gevormd werd na 24 Augustus en in 13° 2 loofbladen.

Er is dus wel een groot verschil met de bloemvorming in de trekproeven van Iris Imperator, die bij het bereiken van 6 cm nog maar pas begonnen is (zie Med. N°. 57, tabel 1). Ook de 2e bloem van Wedgwood was bij het bereiken van 6 cm neuslengte reeds vrij ver ontwikkeld, n.l. in stadium III tot IV; 3 weken later was in 15° deze 2e bloem geheel klaar, in de andere temperaturen in stadium V tot VI. In onze trekproeven ontplooiden deze tweede bloemen zich slechts in enkele gevallen, in totaal bij 23 van de 480 bollen. Verder kwam eenmaal een z.g. derde bloem voor in den oksel van het hoogste loofblad, de tweede bloem ontplooide zich daarbij echter niet. Bij het gefixeerde materiaal werd deze z.g. derde bloem (zie Med. N°. 62 blz. 810) vrij vaak aangetroffen, vooral in het materiaal, dat 3 weken na 6 cm gefixeerd was.

De bloei van de in 1939 opgezette proeven was dus zeer bevredigend, n.l. 90 tot 100 %, behalve in één proef (de snelste) van tabel 8 (80 %). Voor een deel is dit zeker te danken aan de betere voeding gedurende de ontwikkeling in de kistjes. Maar het feit dat ook de proef met zuiver duinzand (tabel 4) dit jaar 100 % bloei gaf, tegenover 70 % in 1937 (tabel 3, 1e regel), wijst er reeds op, dat 1939 een bijzonder gunstig bloei-jaar is geweest. Waaraan dit toe te schrijven is, weten wij niet. Wij hopen echter door voortzetting van deze trekproeven tenslotte te kunnen vaststellen, welke factoren hierbij een rol spelen. Wij denken vooral aan de temperatuur in den grond gedurende de laatste weken vóór het rooien. De gegevens, die wij tot nu toe daarover verzameld hebben, blijken voor conclusies nog niet voldoende.

Samenvatting der resultaten en advies voor de practijk.

1. Voor goeden en vroegen bloei van Iris Wedgwood is een *warme voorbehandeling noodzakelijk*. Men kieze daarvoor 3 weken 31° of 5 weken 23° à 28° C.

2. De *snelste bloei* wordt bereikt door *bewaren en planten in 13° na de voorbehandeling van 3 weken 31°*. Zoodra alle spruiten 6 cm buiten den bol steken, kan overgebracht worden naar een matig warme kas van 15°. Beter is het nog 3 weken te wachten, omdat daardoor de kans op verdroging minder wordt, hoewel men iets in snelheid verliest.

3. Overbrenging naar een warme kas van 17° geeft nog enkele dagen vervroeging van den bloei; dit overbrengen geschiedde in onze proeven pas bij het zichtbaar-worden van het 5e loofblad, maar het ging gepaard met een vermindering van het bloei-percentage van 90 à 100 % tot 80 %.

4. Bewaren en planten in 11° in plaats van in 13° na de voorbehandeling van 3 weken 31° geeft slechts enkele dagen verlating van den bloei.

Bewaren en planten in 9° vertraagt den bloei van Wedgwood veel sterker. De kans op verdrogen der bloemen is in 9° en 11° minder dan in 13°.

5. Na de warme voorbehandeling kunnen de bollen zonder bezwaar nog gedurende 6 weken *droog bewaard worden* bij 13° (resp. 11° of 9°). Dit vertraagt den bloei slechts weinig en geeft verschillende voordeelen voor de practijk.

6. Uitplanten van de bollen in kistjes met *voedzamen grond vermengd met gravel* (verhouding 4 : 1) gaf in onze proeven mooiere resultaten dan uitplanten in zuiver zand.

Wageningen, Mei 1940.

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SUMMARY.

Early flowering of Dutch Irises var. Wedgwood.

Iris Wedgwood, originated from crosses of the Moroccan Iris tingitana, is one of the most favourable Dutch Irises by the colour of its flower and its tendency to early flowering.

By various experiments it could first of all be stated that the flower-origination of Wedgwood is favourably influenced by a hot preliminary treatment, e.g. at 23° or 28° during 3 to 5 weeks (see table 1).

After the preliminary treatment the bulbs are planted at a lower temperature. The quickest extension was found after planting in 13° (see table 2), after this temperature also the earliest flowering occurred. In the first experiments the plants were conveyed to a greenhouse of 15° when they had reached a nose-length of 6 cm. Later the low tempe-

rature treatment was continued for a few weeks, without yielding better results (table 3). This was confirmed by experiments in 1939/40, in which the plants were conveyed to a greenhouse of 15° as soon as they were 6 cm long (table 5), after 10 days (table 6) and after 3 weeks (table 7). By fixations it could be stated that when a nose-length of 6 cm has been reached, the flower of Wedgwood is pretty well finished, except the carpels, which have not yet formed then (see table 9). Here also the flower-origination of Wedgwood proved to be earlier than that of Imperator (cf. also Comm. 57 and 62).

The method usual in practice, by which the bulbs are not at once planted after the hot preliminary treatment, but are first kept dry for a few weeks at the lower temperature, proved satisfactory also in our experiments. Flowering is not appreciably retarded by it (see table 4). Also GRIFFITHS (1936) indicates a hot preliminary treatment at 80° F. ($26\frac{1}{2}$° C.) during the month of July (in America the bulbs are already dug in the beginning of July), after which they are kept at 50° F. (9° C.) during August and not planted until the beginning of September.

The quickest and at the same time good bloom was in our experiments of 1939 obtained after a preliminary treatment of 3 weeks at 31°, followed by 6 weeks at 13° and then planting at 13°; when a length of 6 cm was reached the boxes were removed to a greenhouse of 15°, where flowering started on January 10, 1940 (see table 5). Flowering started sooner still with a batch which, after the same preliminary treatment, was only put in the greenhouse of 15° 3 weeks after attaining 6 cm, and 3 days later, when the 5th foliage-leaf became visible, in the greenhouse of 17°. Here flowering started already on January 4 (see table 8), but 4 of the 20 flowers shrivelled.

The very good results of the series of experiments in 1939/40 (table 5—8) are ascribed to the use of a nourishing soil, mixed with gravel. It should be borne in mind, however, that this year was a very favourable flowering-year generally.

Anatomy. — *The anatomy of the mammary gland in mice with regard to the degree of its disposition for cancer* [1]). By P. J. VAN GULIK and R. KORTEWEG. (Communicated by Prof. M. W. WOERDEMAN.)

(Communicated at the meeting of June 29, 1940.)

I. *Introduction.*

In 1933 LITTLE c.s. in America and KORTEWEG in the Netherlands proved that the disposition for cancer of the mammary gland in female mice is largely determined by an influence (a so-called extrachromosomal factor) on the young ones, emanating from the mother, but not from the father. (An indication in this direction which, owing to the, at that time still imperfect technique, could not be elaborated, had already been given by LATHROP and LOEB as early as 1918).

As in our breeding-colony the father also invariably stays with the young ones, only the following influences, exclusively working from the mother's side, are taken into consideration:

1. the composition of the cytoplasma of the ovum;
2. the influence emanating from the mother-animal on the young ones while staying in utero;
3. the composition of the milk sucked by the young ones.

That however also a purely genetic (chromosomal) factor makes its influence felt in determining the degree of this disposition was emphatically argued by KORTEWEG in 1936 (1).

In 1936 BITTNER demonstrated that the extrachromosomal factor might be identical with the influence emanating from the milk on the young ones (2). As soon as possible after their birth BITTNER had separated the young ones of a cancer-strain from their mothers, and had put them with suckling mice of a non-cancer-strain. Later on most of these foster-nursed mice did not get cancer in spite of all expectations. Similar experiments on a larger scale were made by BITTNER, ANDERVONT and KORTEWEG. This paper relates to these experiments and to an extensive anatomical investigation of the mice involved.

II. *Materials.*

Our experiments were made on specimens of three different inbred strains of mice and on hybrids of these strains:

1) From the Laboratory of the Antoni van Leeuwenhoekhuis (Netherlands Institute for Cancer Research).

1. the dilute brown strain MURRAY—LITTLE (called K-strain by us);
2. the black strain C 57 LITTLE (by us called G-strain);
3. the O 20 Leeuwenhoekhuis-strain (bred by us for 20 generations by means of brother-to-sister mating, and called by us O-strain).

We exclusively used virginal female mice (in male mice cancer of the mammary gland practically does not occur). By "cancer" we mean in this paper "cancer of the mammary gland".

In the K-strain 85 %, in the O-strain 1 % of the virginal females get cancer of the mammary gland. In the G-strain this type of cancer was only once found in several hundreds of animals.

III. *Influence of the milk and of the genetic composition on the frequency of cancer of the mammary gland.*

A. *Influence of the milk.*

Table I shows our results with mice nursed by their own mother or foster-nursed. Of K-females, nursed by their own mother, 85 % get cancer

TABLE I.

Type of mice	Normal or controls			Foster-nursed		
	Number of mice	Number developing breast cancer	Tumor incidence (percent)	Number of mice	Number developing breast cancer	Tumor incidence (percent)
K	168	142	85 ±2.8	110	10	9 ± 2.7
G	194	0	0	67	9	13 ± 4.1
F_1 K ♀ × G ♂	335	232	69 ±2.5	47	2	4 ± 2.9
F_1 G ♀ × K ♂	722	11	1.5±0.4	91	42	46 ± 5.2
O_{20}	272	3	1 ±1.6	—	—	—
π_{10}	101	1	1	—	—	—
$-\pi_{10}$	95	1	1	—	—	—

later on; if K-females are brought up by a foster-mother of the non-cancer strain G, this percentage falls to 9 %. Whilst the mother-nursed G-females practically never get cancer later on, 13 % of them get cancer when brought up on the milk of a K-strain foster-mother.

Of the F_1-hybrids between these strains, whose mothers belong to the K-strain, 69 % respectively 4 % get cancer, according to their having been suckled by their own mother or by a G-foster-mother. Of the reciprocal F_1 hybrids bred by mothers belonging to the G-strain, 1½ % get cancer, and 46 % of those fostered by a K-foster-mother.

These results obtained by us, mainly corroborate BITTNERs (3) and ANDERVONTs (4), and they convincingly prove the very great influence

which the milk, on which the mice are bred, exercises on the degree of their disposition for cancer of the mammary gland.

B. *Influence of the genetic composition.*

When comparing the normal K-females with the F_1-females K♀×G♂, which two types of mice had a K-mother, we see that the cancer percentage is significantly higher when also the father is a K-animal (85 % ± 2,8) than when the latter belongs to the G-non-cancer strain (69 % ± 2,5). Here the influence of the father on the disposition for cancer is proved; this influence cannot be but a purely genetic one. The percentages we found in our foster-nursed females of the same groups: 9 % ± 2,7 and 4 % ± 2,9 are not significantly different, although pointing in the same direction.

On comparing the fostered G-females with the fostered F_1-females whose mothers belonged to the G-strain, we found 13 % ± 4,1 resp. 46 % ± 5,2, a statistically significant difference, from which also the influence of the paternal, consequently purely genetic factor, appears.

In backcrosses from one strain to the other we found the following facts:

Starting from K-females and regularly crossing them and their female descendants in the succeeding generations with G-males, it appeared that of 101 consequently 11 times backcrossed females (by us called π_{10}) only one got cancer of the mammary gland. After having been backcrossed so repeatedly these mice have practically got the genetic composition of G-mice. The influence of the extrachromosomal factor on the disposition for cancer has been entirely lost in them.

In backcrossings of G-females to the K-strain, only one out of the 95, eleven times backcrossed, females (called by us $-\pi_{10}$) appeared to have got cancer of the mammary gland. Although these mice have practically obtained the genetic composition of the cancer-strain, yet the disposition for cancer of the mammary gland is very low in them.

The genetic composition for the disposition for cancer of the mammary gland is apparently of little importance if not one or more extrachromosomal factors are active too. In case of presence of these extrachromosomal factors there is no doubt about the significance of the genetic composition.

In the K-strain in which the extrachromosomal factor and the chromosomal factor exercise their influence together again and again, the disposition for cancer of the mammary gland, as a result of these factors, has remained very high for more than 80 generations.

In backcrossing G-females to the K-strain, the disposition remains low, even after 11 backcrossings (our $-\pi_{10}$ animals), though genetically these animals practically belong to the K-strain. From this we may conclude that the extrachromosomal factor is not in the last instance a product of the genetic composition, but that the latter is independent of it.

In backcrossing K-females to the G-strain the disposition has, after 11 backcrossings (our π_{10} animals) become very low, even lower than in

our G-mice fostered by a K-female. From this we may conclude that the extrachromosomal factor becomes inactive after a number of generations when the chromosomal factor is not present at the same time.

IV. *Architecture of the mammary gland in our mice.*

After the method of VINTEMBERGER (5), slightly modified by the authors, the second mammary gland on one side was dissected, stained in hemalum and mounted in balsam of Canada. (The corresponding second mammary gland on the other side and the inguinal mammary glands were embedded for histological examination. The authors will report on this later on). Of every dissected mammary gland an enlarged drawing was made with a projection-apparatus.

A. *Normal mice.*

Fig. 1 presents the mammary glands of a G-strain and of a K-strain mouse of 8 months old. When killed both mice were in the same stage of the di-estrus. The architecture of the mammary gland of the G-strain

Fig. 1. Drawing of the projected second mammary gland of normal mice. Some ducts penetrate pads of fat, which are indicated by a dotted line. In these pads of fat the milk ducts are not dissected.
a = mammary gland of a G-strain mouse; resembles a tree in wintertime.
b = mammary gland of a K-strain mouse; resembles a tree in springtime.

mouse recalls the picture of a tree in winter; the mammary gland of the K-strain mouse resembles a budding tree in spring. The primary duct in the G-strain mouse (fig. 2) merely shows widening and buds in the second half, starting from the teat, whereas the beginnings of the secondary ducts show buds nor widening hardly anywhere. In contrast with this we find in the K-strain mouse (fig. 3) similar widening and buds along the entire length of the primary duct and also at the beginning of the secondary ducts.

Fig. 4 presents a drawing of the second mammary gland of a nine months old O-strain mouse in the di-estrus stage. The architecture of its gland-tree is between that of the G-strain and the K-strain. The primary duct is longer and shows many small buds along its entire length; the beginning of the secondary ducts is nearly devoid of similar buds.

Fig. 4. Second mammary gland of a normal O-strain mouse.

In a great number of specimens of these three strains the above anatomical characteristics appeared to be constantly present. The characteristic strain-differences of the primary duct were already typically present in new-born mice.

Our study being in progress, GARDNER and his co-workers (6) published an interesting paper. They examined the second mammary gland of non-virginal mice of different strains. The authors mentioned noticed in the mammary gland of cancer-strain mice (the same strain which we examined), when the animals were getting older, nodules of hyperplastic mammary tissue. The frequency of these nodules, which in a sense may be considered a preliminary stage of cancer, increases as the mice have advanced further into the "cancer-age". On the basis of our material we can, as far as virginal mice are concerned, fully confirm GARDNERs discovery. (Fig. 5.)

GARDNER c.s. did not notice the differences in architecture of the mammary gland in the various strains; yet these differences are distinctly visible in the photos reproduced by them.

In the normal F_1-hybrids of which the mother belonged to the K-strain, the gland-tree principally represents the K-type, but it also slightly resembles the G-type (Fig. 6 b). In the normal F_1-hybrids of which the mother belonged to the G-strain the gland-tree shows the G-type, slightly modified in the direction of the K-type (Fig. 6 a).

In these F_1-hybrids the primary duct represents the type characteristic of the strain to which the mother belonged [1]).

B. *Foster-nursed mice.*

In the K-females fostered by G-females (Fig. 7 b), the primary duct shows the normal K-type; the gland-tree however possesses the G-aspect.

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[1]) In both species of backcrossed mice (π_{10} and—π_{10}) the gland-tree represents the G-type, in π_{10} mice, started from a K-female, slightly modified in the direction of the K-type. The study of these mice is still in progress.

In the G-females fostered by K-females (Fig. 7 a) the primary duct presents the normal G-type, but in the gland-tree a very remarkable

Fig. 6. Second mammary gland of normal F_1 hybrids.
a = F_1 G♀ × K♂ .; b = F_1 K♀ × G♂ .

Fig. 7. Second mammary gland of foster-nursed mice.
a = G-female foster-nursed by K-female. K-branches inoculated on a G-tree. In these inoculated K-branches nodules (indicated by hatched areas).
b = K-female foster-nursed by G-female. In this gland-tree the milkducts are all alike.

change is to be observed. On the whole the architecture of the gland-tree is of the G-type, but at the same time some ducts spring from the ducts, which, in architecture are identical with that of the K-type. It looks as if here ducts of the K-type had been inoculated on a gland-tree of the G-type (Fig. 8). In these same inoculated milkducts we found the nodules described by GARDNER.

In the fostered F_1-hybrids the primary duct presents the type belonging to the mother (Fig. 9). In both types of hybrids the gland-tree is mainly of the G-type, but in the G-females fostered by K-females we again find

Fig. 9. Second mammary gland of foster-nursed F_1 hybrids.
a = F_1 G♀ × K♂ foster-nursed by K-female. Here again inoculating of K-milkducts on a G-gland-tree. In these grafted milkducts formation of nodules.
b = F_1 K♀ × G♂ foster-nursed by G-female. No inoculating at all.

a similar "inoculating" of milkducts of the K-type on a gland-tree of the G-type (Fig. 10).

In table II we have collected the various data.

The first column gives the types of mice, the second column the manner

TABLE II.

1	2	3	4	5	6	7	8	9
Type of mice	Nursed by:	Genetic formula	Extrachromosomal factors:			Architecture of mamm. gland		Tumor incidence
			Ovular plasma	Uterine influence	Milk-factor	Primary duct	Gland-tree	
K	Own mother	KK	K	K	K	K	K	85 %
G	„ „	GG	G	G	G	G	G	0 %
F_1 K ♀ × G ♂	„ „	KG	K	K	K	K	K → G	69 %
F_1 G ♀ × K ♂	„ „	GK	G	G	G	G	G → K	1.5 %
K	G foster-mother	KK	K	K	G	K	G	9 %
G	K-foster-mother	GG	G	G	K	G	G + K	13 %
F_1 K ♀ × G ♂	G-foster-mother	KG	K	K	G	K	G → K	4 %
F_1 G ♀ × K ♂	K-foster-mother	GK	G	G	K	G	G + K	46 %

of nursing, the third the genetic formula, in other words the chromosomal factors which may be of importance for the degree of the disposition. The extrachromosomal factors: plasma of the ovum, intra-uterine influence and kind of the milk are given in the 4th, 5th and 6th columns, K resp. G meaning that the factor relating to it, originates in a K- resp. G-mouse.

In the 7th and 8th column we give the architecture of the mammary gland: in the 7th column the architecture of the primary duct with the beginning of the first ramification and in the 8th column that of the gland-tree. In the latter case the first letter indicates the principal type; $G \rightarrow K$ meaning G-type modified slightly in the direction of the K-type; $G + K$ means G-type, in which as a foreign element, sharply distinguished from it, some milkducts of the K-type are found. In the last column the frequency of cancer of the mammary gland is given for each of the various types of mice.

From this table it appears that the primary duct presents the pure type of the maternal animal; the reciprocal F_1-hybrids possess a different type, although their genetic formulae are identical. From the fact that the nature of the milk is immaterial for the architecture of the primary duct (foster-nursing makes no difference) it follows that the influence of plasma and uterus, or one of them must be responsible for this architecture. (As these two factors, according to the method applied by us, are invariably both derived from the same animal, the significance of each of these factors cannot be settled by us).

The architecture of the gland-tree in pure K-mice differs from that of F_1-hybrids K♀ × G♂. In these two the genetic composition differs, whilst the nature of the extrachromosomal factors is the same. A similar difference in architecture exists between pure G-mice and the F_1-hybrids G♀ × K♂. From this it follows that the architecture of the gland-tree is determined, at least partly, by purely genetic influences.

From our table it appears however that the influence of the extra-chromosomal factors on the architecture of the gland-tree is of much greater importance: in K-mice, fostered by G-females, the K-type to be expected, under the influence of the G-milk, has been altered into the G-type. Only in case all three extrachromosomal factors are derived from a K-mouse, does the gland-tree show a genuine K-type: if only the milk or only plasma + uterine influence of the K-animal have been at work, the gland-tree shows a — possibly more or less modified — G-type (compare normal K with fostered K, normal F_1 K♀ × G♂ with fostered F_1 K♀ × G♂ and normal F_1 K♀ × G♂ with fostered F_1 G♀ × K♂). Both the kind of milk and the nature of plasma- and uterusfactor consequently influence the architecture of the gland-tree.

In the two types foster-nursed by a K-mouse (G by K and F_1 G♀ × K♂ by K) inoculation of K-ducts on the G-gland-tree has taken place, as a distinctly foreign element.

V. *The anatomical architecture of the mammary gland as an explanation for the degree of its disposition for cancer.*

From table II it appears that the only gland-tree showing the genuine K-type (normal K-females) possesses the highest disposition for cancer by far. Next comes, concerning the degree of this disposition, the only other gland-tree of the K-type already slightly modified however in the direction of the G-type (normal F_1 K♀ × G♂ hybrids). Hereupon follow both gland-trees of the G-type which show inoculated genuine K-ducts (G-females foster-nursed by K-females; F_1 G♀ × K♂ hybrids foster-nursed by K-females). The disposition of the G-gland-trees, which have been slightly modified in the direction of the K-type but still possess no genuine K-type ducts is still lower (F_1 K♀ × G♂ hybrids foster-nursed by G-females; normal F_1 G♀ × K♂ hybrids).

The disposition of the gland-tree of the genuine G-type is extremely low. (In K-females fostered by G-females, with 9 % cancer, we found a gland-tree of the G-type. This apparent contradiction may be explained by the fact that of these mice only a very small number have been examined).

From our material it most convincingly appears that the degree of the disposition for cancer of the mammary gland is closely connected with the anatomical architecture of this organ.

VI. *Summary.*

The factors determining the degree of disposition for cancer of the mammary gland in the mouse are discussed on the basis of the result of extensive breeding experiments. In a great number of mice the anatomical architecture of the mammary gland was examined. The relation existing between this architecture and the disposition for cancer of the mammary gland is discussed.

VII. *Conclusions.*

1. There exist typical differences in architecture of the mammary gland between the mice-strains examined by the authors.
2. This architecture is influenced both by genetic and extrachromosomal factors.
3. The architecture of the primary duct is determined by the plasma- and (or) uterus-factor.
4. The architecture of the gland-tree is determined both by the genetic and by the extrachromosomal factors: milk, plasma- and uterusfactor.
5. There is a strong positive correlation between the architecture of the mammary gland in different types of mice with regard to the degree of its disposition for cancer.

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Neurology. — *Further observations about the cervical form of position-nystagmus and its anatomical base.* By A. BIEMOND, chef-de-clinique of the Neurological Clinic, Wilhelminagasthuis, Amsterdam. (From the Neurological Clinic, Director Prof. B. BROUWER, and the Otological Clinic, Director Prof. A. DE KLEYN). (Communicated by Prof. B. BROUWER).

(Communicated at the meeting of June 29, 1940.)

In 1939 the author of this article published a communication on a new form of experimental position-nystagmus in rabbits and its clinical significance for man [1]). The nystagmus could be elicited in 8 of the 10 cases by cutting the second cervical posterior root of the animals. This nystagmus which was described in detail, occurred immediately after the cutting, appeared only in both lateral positions, was nearly always of an associated character and could be elicited only during a restricted time (½ to ¾ of an hour). In the same article three clinical observations were recorded: the patients, suffering from a neuritis, resp. radiculitis cervico-brachialis, developed in the course of this disease a severe dizziness. Examination revealed a typical position-nystagmus, whereas the vestibular reactivity was normal. In these cases the dizziness and the position-nystagmus disappeared almost simultaneously with the cure of the neuritic symptoms.

It was taken for granted that here the position-nystagmus was of peripheral (cervical) origin. This idea was strongly supported by the experi ments mentioned above.

The author pointed to the necessity of enlarging the experimental investigation, partly by cutting the lower cervical roots, partly by anatomical examination of the retrograde degeneration in the spinal cord and in the medulla oblongata.

In the following pages this investigation, performed in the laboratories of Prof. DE KLEYN and Prof. BROUWER, is described. After that some new observations of cervical position-nystagmus in man are communicated.

Cutting of the 3rd. resp. 4th or 5th cervical posterior root in rabbits.

This operation presented great technical difficulties as the third and following cervical roots ramify directly after its emergence from the intervertebral foramen. Therefore cutting of these roots can only be performed after opening of the vertebral canal. However, a severe venous hemorrhage which is usually fatal, always follows: in the first series of experiments

(cutting of the third cervical root) all animals died before the roots were cut. The big amount of experimental animals, placed at our disposal together with the continuous improvement of the operative technic, enabled us, however, to examine a few rabbits in the desired way. The results were as follows:

In 3 experiments we were able to cut the third left root. All three animals showed a distinct position-nystagmus, which especially occurred when the head of the animal was turned to the right. At first the nystagmus was of a rotatory character (whereby in the quick phase the upper pole of the left eye was turned in temporal direction) but soon passed off into a horizontal nystagmus. Also in the left lateral position a distinct nystagmus could be seen but less marked than in the right lateral position. All three animals died some minutes after cutting of the cervical root.

In another series of 3 rabbits we could cut the fourth left cervical root. In these three experiments a typical position-nystagmus occurred with the same symptoms as described above and also more marked in the right lateral position. Here also the animals could be kept alive only a few minutes. In two cases the fifth cervical root was cut: here not even the slightest sign of a position-nystagmus developed, no more than in one case where the sixth cervical root was cut.

Further experiments of cutting the 2nd cervical root.

As mentioned above, in our first paper 10 experiments of cutting the 2nd cervical root were recorded, of which in 8 cases a typical position-nystagmus was observed. We thought it necessary to investigate why in two cases the cutting had a negative result. For this investigation we made use of a larger series of rabbits (25) in which the same operation was performed. Special attention was given to moments which possibly could have a positive or negative influence. From this new series 18 rabbits developed a distinct position-nystagmus. We had the impression that a quickly performed operation could easily damage the 2nd cervical root or one of its principal ramifications before the cutting was done. In this way the experiment could possibly be influenced unfavorably and this could be responsible for the absence of the position-nystagmus in the minor part of the cases. In fact it could be demonstrated that 5 rabbits, in which the 2nd cervical root was first freed carefully under conscientious evitation of any premature irritation, and thereafter cut, *all* developed a distinct position-nystagmus.

Summarizing we can conclude that extravertebral cutting of the 2nd cervical posterior root in rabbits causes in the large majority of cases (31 of 40 experiments) a typical position-nystagmus; this nystagmus was elicited in *all* cases (two series of 3 experiments) by intravertebral cutting of the 3rd or 4th cervical posterior root. It seems impossible to produce this phenomenon by cutting the lower cervical roots.

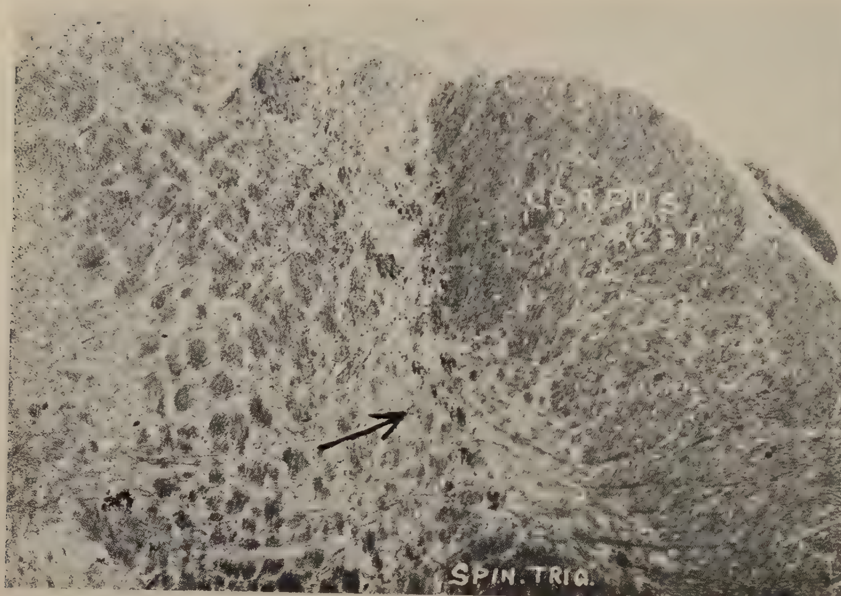

Fig. IV

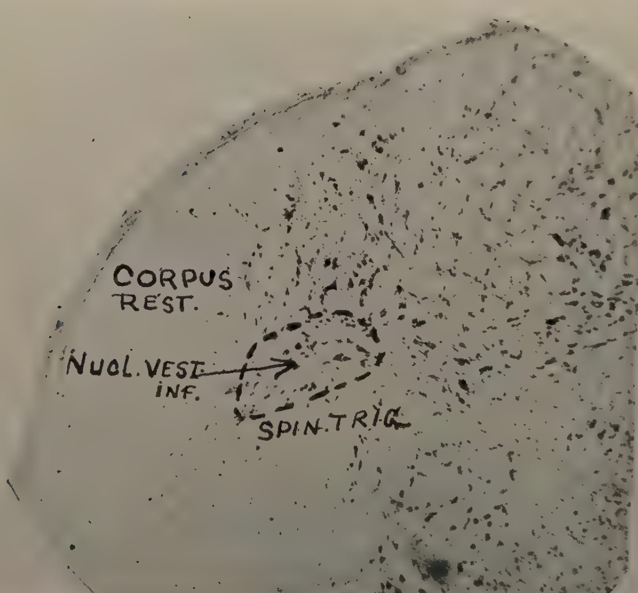

Fig. V. Nucleus vestibularis inferior (dotted line) in a normal Nissl-coupe (rabbit).

Anatomical research.

Two rabbits in which the left second cervical posterior root was cut, were killed after two weeks: the cervical cord and the medulla oblongata were cut serially and stained by the Marchi-method. At the same time we disposed of serial sections of the same parts of the central nervous system of three rabbits in which respectively C III, C IV and C V were cut. These last series originated of previous experiments of dr. TEN CATE, performed for Prof. BROUWER for other purposes. These sections could be excellently used for our present investigation. In discussing the C II sections the Marchi preparations reveal that in both cases a completely specific secondary degeneration developed, which can be followed systematically from the cervical cord to the medulla oblongata. At a level under the cutting of the root, the Marchi granules occupy a certain area in the funiculus of Burdach. Here it is noticable that the granules lay not closely packed but are scattered in a rather large cross-area. It is evident that here a degeneration in the descending fibres of the posterior root exists. On the level of the cut posterior root, the degeneration of this root and the localized degeneration of the posterior columns forms a single unintermittent area of Marchi granules. Above the level of the cut root the Marchi granules, which here are closely packed and form a circumscript area, take a very definite position in the lateral part of the funiculus of Burdach (fig. I). At a more higher level in the series the pyramidal crossing is seen and at the same time the nuclei of Goll are found. It is evident that the degeneration of Marchi does not have any contact with the nuclei of Goll. At a still higher level we can see the hypoglossus nucleus and hypoglossus roots; here also the vagus nucleus becomes visible. On this level the nucleus of Burdach is distinctly seen laterally of the nucleus of Goll. The area of Marchi granules changes its form; it now has the appearance of a comma with its convexity directed to medial. The biggest (upper) part of the comma comes directly into contact with the nucleus of Burdach and, in following sections, begins to merge itself gradually into it (fig. II and III). The smaller lower part of the comma now enters into contact with the nucleus of von Monakow, which appears laterally under the nucleus of Burdach. Some degenerated fibres also pass into the nucleus of von Monakow. On a much higher level, where the nuclei of Goll, Burdach and von Monakow disappear, a small area of Marchi granules is still present, which merges into the corner formed between the spinal trigeminus root and the corpus restiforme. Comparition with Nissl-coupes of different levels, just before and after the end of the nuclei of the posterior column, show that the Marchi-granules (at least a small amount) prolong further than the last rest of the nuclei of the posterior column. It also appears that the most proximal part of the degenerated fibres merge into an area which is occupied by the *lower vestibular nucleus* (nucleus vestibularis inferior, nucleus radix descendens VIII) (fig. IV and V).

Examination of the other series (resp. cutting of C III, C IV or C V) reveals the following. Also here a degeneration in the posterior column, occupying a more or less circumscript area, could be followed regularly from below to above. It was evident that here too part of the degenerated fibres merge gradually into the nucleus of BURDACH and that another part passed into the nucleus of VON MONAKOW. Here also, at least in the series where C III or C IV were cut, a thin band of fibres was seen which extended above the level of the last rests of the nucleus of the posterior column and passed into the above described area of the nucleus vestibularis inferior. Here attention must be drawn to the fact that the mentioned area of fibres in which C IV was cut, appeared to be of minor caliber than in the series where C III was cut. In the C V-series it could not be concluded with sufficient certainty that an extention of the degeneration above the level of the nuclei of BURDACH and VON MONAKOW was present.

From this anatomical examination it has become evident that cutting of C II, resp. of C III or C IV in rabbits, causes a secondary degeneration in the homolateral posterior column, which not only merges into the nuclei of BURDACH and VON MONAKOW, but at the same time partly passes into the nucleus vestibularis inferior, which, as known, shows a gradual transition into the nucleus of DEITERS which is found more laterally *).

The above described new form of experimental position-nystagmus in rabbits, caused by cutting resp. of C II, C III or C IV thus gets an anatomical base. It was namely possible to conclude that the same roots send a certain contingent of their fibres to the lower vestibular nucleus. It may be considered reasonable that cutting of these roots causes in this nucleus-area a stimulation-condition resulting in a position-nystagmus. The fact that this experimental position-nystagmus is only of short duration completely corresponds to this conception. It is, however, still inexplicable why this experimental position-nystagmus asserted itself more in the contralateral than in the homolateral position.

Significance for the neurological clinic.

The general clinical significance of position-nystagmus is discussed briefly in our last publication. A more detailed survey is given in a recent summarizing article of MEYER zum GOTTESBERGE [4]). However, in this paper the new cervical form of position-nystagmus is not mentioned.

The above described, experimentally and anatomically founded, form of

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*) How far this also holds true for other mammals must further be investigated. FERRARO and BARRERA [2]) who examined the secondary degeneration after lesion of the cervical posterior roots in macacus rhesus, saw that this degeneration ended only in the nucleus cuneatus and in the nucleus of Von Monakow. The ventrolateral part of the last nucleus, however, in which especially the higher cervical roots should end, nearly comes into contact with the nucleus vestibularis inferior. KAPPERS, HUBER and CROSBY have pointed to the fact that the nucleus cuneatus externus receives descending vestibular fibres but do not mention a connection in retrograde direction [3]).

position-nystagmus will be interpreted as the *cervical* form of position-nystagmus. As far as known it has special clinical significance in those cases of neuritis (resp. radiculitis) cervico-brachialis, where dizziness and position-nystagmus are associated. As mentioned above, in our first paper three of such cases are recorded. Last year we were able to observe again two patients suffering from this combined affection. Both were middle-aged women. In the first case a left-sided neuritis brachialis with position-nystagmus to the left was present when the patient was brought in the left lateral position. The second case was a right-side neuritis brachialis with position-nystagmus to the right, present in the right lateral position. In both cases the caloric vestibular reactions were normal. All symptoms disappeared completely under the usual treatment of the neuritis.

It may be expected that extra- and intrameduallar processes, especially tumors, will give rise to a typical position-nystagmus. However, as the systematical investigation on position-nystagmus is of recent date, further particulars are not available.

In literature some observations are recorded in which attention is drawn to the occurrence of nystagmus and dizziness in processes of the cervical cord (HELSMOORTEL and v. BOGAERT [5]). A satisfactory explanation could not be given.

Position-nystagmus after cutting of the 2nd and 3rd cervical root in man.

We are able to communicate one observation in man, which may be considered as a physiological equivalent to the experiment in animal.

A man, 25 years of age, suffering for 9 months of a severe torticollis to the right, was admitted to the Neurological Clinic on November 3, 1939. Neither medical, nor electrical therapy had any effect: this was the reason why on April 19, 1940 laminectomy with intradural cutting of the left 2nd and 3rd cervical posterior root was performed. Immediately after this operation (which was done with local anesthesia) a marked nystagmus to the left developed if the patient turned his head to the right or to the left. Thus here a typical position-nystagmus was present which existed for at least 20 minutes. During this period the surgeon finished the operation in the usual way. After the operation the patient was examined in the left and right lateral position for about 40 minutes. It now appeared that only in the right lateral position a slight nystagmus was present to the left; in the left lateral position this nystagmus was absent.

One hour after the cutting the position-nystagmus could no longer be elicited.

The torticollis was greatly improved by the operation.

Summary and conclusions.

1. Cutting of the 2nd, 3rd or 4th cervical posterior root in rabbits elicited a position-nystagmus. Also in man this phenomenon could be seen after cutting of the 2nd and 3rd cervical posterior root.

2. Examination of the Marchi-degeneration in rabbits, in which such cutting of a posterior root was performed, revealed that part of the fibres of the 2nd, 3rd and 4th cervical posterior root terminate in the vestibular nucleus.

3. These physiological and anatomical data may form the base for a new neurological syndrome consisting of: cervical lesion of the posterior root with position-nystagmus and position-dizziness. Of this syndrome some cases are recorded.

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Medicine. — *Is thyrotropic hormone consumed, when acting upon the thyroid gland?* By F. GLASER and J. A. COHEN. (Communicated by Prof. J. VAN DER HOEVE.)

(Communicated at the meeting of June 29, 1940.)

Concerning the nature of hormonal activity great uncertainty still prevails. There are data which are indicative of a *consumption* (decay, chemical linking, adsorption, accumulation) at the seat of activity (e.g. the observations of GAILLARD and DE JONGH on the uterus explantates in vitro) (1).

Other experiments would seem to point rather in the direction of a hormonal activity, solely dependent upon the *concentration* of the hormones in the tissue fluids and thus independent of the quantity of reacting tissue. A certain quantity of testosterone, for instance, has the same effect on the *intact* male genital tract as upon the remainder after partial removal. The same holds good for the effect of oestrone upon the female tractus (HERZ c.s.) (2).

We endeavoured to solve this problem by an investigation upon adult hypophysectomized rats, a number of which also underwent extirpation of one thyroid, in order to ascertain in how far there was any difference in both groups in the activity of the thyroid gland (judged from the histological aspect) after injection of a certain dose of thyrotropic hormone. Thyrotropic hormone was chosen, since apart from the thyroid gland, few seats of activity are known. (This not being the case for the influence of oestrone and testosterone on the female and male genital tract respectively, the correctness of HERZ' conclusion, quoted above, might be questioned.)

Technique:

The hypophysis of adult male rats was removed. In some animals of each series, simultaneously experimented upon, unilateral thyroidectomy was performed at the same time. Six days later subcutaneous injections with thyrotropic hormone were begun (10 U Ambinon Organon per rat daily). The day after the last injection, the animals were autopsied and the activity of the thyroid gland (or glands) was estimated histologically according to the criteria of HEYL and LAQUEUR (3). Only those animals are recorded below, the sellae of which had proved to contain no pituitary remainders.

Results:

Group I (Control group, not injected, autopsy 7 days after the operation).

Number of animal	Weight at operation	Weight at autopsy	Thyroid activity
A. *Hypophysectomy.*			
B 6122	169	160	P
B 6123	150	143	P
B 6124	163	154	P
B. *Hypophysectomy and unilateral thyroidectomy.*			
B 6125	167	160	P
B 6126	153	152	P
B 6127	146	141	P

No difference between groups A and B.

Group II. Six days after the operation the animals were injected subcutaneously with 10 U thyr. hormone daily, on the 6th and 7th day after operation. Autopsy the 8th day.

Number of animal	Weight at operation	Weight at autopsy	Thyroid activity
A. *Hypophysectomy.*			
B 6110	136	124	r
B 6111	161	148	s
B 6112	156	154	s
B 6114	176	170	r—s
B 6115	183	171	r—s
B. *Hypophysectomy and unilateral thyroidectomy.*			
B 6116	160	147	r
B 6117	154	152	r—s
B 6118	176	156	r—s
B 6119	161	150	s—t
B 6120	171	158	r—s
B 6121	143	116	r—s

No significant difference between groups A and B.

Group III. From at least 6 days after the operation daily subcutaneous injection with 10 U thyrotropic hormone, for 7 days.

Number of animal	Weight at operation	Weight at autopsy	Thyroid activity
A. *Hypophysectomy.*			
6891	190	158	r
6892	150	130	r
6893	170	142	r
6895	156	148	s
6898	175	164	q
5411	263	220	q
5418	222	190	r
5419	182	165	s
B. *Hypophysectomy and unilateral thyroidectomy.*			
B 6889	168	138	r
B 6890	150	150	q—r
B 6896	154	134	q—s
B 6897	152	124	q—r ?
B 6899	170	146	q—r
B 6900	194	161	q
B 6901	188	254	r
B 5410	219	200	r—s
B 5413	245	235	r
B 5415	206	184	s
B 5416	194	180	r—s

Again no difference of any importance was observed.

An unequal inhibition of the thyroids by the forming of thyroxine in different quantities, of itself a conceivable source of error, would only have caused difficulty in the event of shown differences; we may therefore pass over this knotty point without comment.

Summary and Conclusions.

The effect of injections of thyrotropic hormone in hypophysectomized adult rats upon the histologically judged activity of the thyroid gland is no greater when one of both thyroids has also been previously extirpated, than in animals in which both glands are present, by a treatment with

thyroid hormone of short duration no more than with one of longer duration.

From this it follows that it is solely the concentration of this hormone in the body which is of importance for its function, and that there is no consumption.

After terminating the above investigation we received the publication of SELYE (4), who, in a similar research as our own came to the same conclusion as regards the action of gonadotropic hormone upon the ovarium.

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Medicine. — *Hypophysis and bloodpicture* VI. *Direct arguments for the blooddemolition-theory.* By G. A. OVERBEEK and P. RUITINGA Jr. (From the department of Pharmacology of the University of Leiden (Holland). Director: Prof. Dr. S. E. DE JONGH). (Communicated by Prof. J. VAN DER HOEVE).

(Communicated at the meeting of June 29, 1940.)

It has become clear from earlier investigations, described in this series of papers, (1. 2. 3. 4. 5.,) that the influence of the hypophysis on the red blood picture is not caused by a direct stimulation of the bone marrow but rather by an augmentation of the processes of erythrocyt-destruction. The arguments given in favour of this theory, though strong enough, always bore an indirect character except the one about the influence on the urobilin excretion. The latter investigation, however, was seriously hindered by technical difficulties, so that the results were less impressive than we hoped them to be. Therefore, we sought and found further direct arguments for our blooddemolition-theory, which will be described in this paper. They are: I. the production of anemia and II. an influence on the bilirubin excretion with the bile after the injection of pituitary extract.

I. If pituitary extracts should contain a substance, promoting blood destruction, one might expect an anemia after treatment with high dosages. In our earlier work we found indications that our injected animals had less erythrocytes than the controls, especially if these controls were injected with boiled pituitary extract, which tends to augment the number of erythrocytes.

After isoelectrical purification of crude alkaline anterior lobe extract (for technical details see the preparation of Y extract, RUITINGA (5)) normal rabbits as well as hypophysectomized rats could be rendered anemic by injection of the new product (table I). This anemia was always accompanied by a strong reticulocytosis. As the number of reticulocytes was much higher than obtained by the injection of a crude extract and often correlated with the seriousness of the anemia (in many cases the anemia became manifest, when the reticulocyte level surmounted 10 %), we think it quite clear that the influence on the reticulocytes must be seen as the consequence of a diminution of the erythrocytes.

One might ask, what could be the cause of the augmented activity of the purified extract. We supposed that the effect was caused by the elimination

TABLE I.

Anemia in rabbits and rats after the injection of pituitary extract.

Number of animal	Erythrocytes mill./mm^3		Reticulocytes ‰		Remarks
	Initial value	Lowest value	Initial value	Highest value	
					Rabbit
D 632	—	1.6	—	320	1×2.5 cc Y extr. for 5 d.
D 633	5.1	3.7	2	60	ibid. 8
—	6.0	4.1	6	6	ibid. 10
—	5.5	4.0	4	40	ibid. 13
					Hyp. ect. rat
B 5484	6.9	4.1	2	220	2×0.5 cc Y-extr. for 14 d.
B 5485	6.9	4.0	8	100	ibid.
B 5487	6.6	5.2	2	120	ibid.
B 6049	6.5	6.2	72	120	ibid.
B 6050	6.6	3.9	48	400	ibid.
B 6051	5.9	5.9	28	72	ibid.
B 6052	7.3	5.7	24	104	ibid.

Note: In some other hypophysectomized rats erythrocyte values were observed of 3.0; 3.7; 4.3 and 3.8 mill./mm^3. As we have no initial values of these animals at our disposition, they are not included in the table, though they may be considered as definitely anemic.

of a factor influencing blood *formation*. Such a factor was described by FLAKS et al. (6). These investigators succeeded in producing a polycythemia in normal rats by the *oral* administration of anterior lobe extract. They never observed any effect after the *injection* of their extracts. As it is not very likely that this substance would show its activity after *oral* administration only, and knowing that crude extracts stimulate blood destruction when *injected*, it becomes probable that the substances described by FLAKS and by us are both present in crude extract. The activity of these crude extracts cannot be very great in either respect, the substances counteracting each others activity.

If this were true, it should be possible to produce a polycythemia by *injecting* the fraction remaining after the preparation of the Y extract.

Although the results were not overwhelming, sometimes a slight polycythemia could be shown after the administration of this extract. In two out of nine rats the erythrocytes increased from 8 million/mm^3 to more than

10 millions/mm^3, this being the level above which FLAKS et al. consider a polycythemia to be present.

In other animals increases to more than nine millions/mm^3 were observed. This result therefore indicates the existence of two substances in the anterior lobe of the hypophysis both stimulating the bone marrow, but acting in a different way. One of them acts in a direct way (FLAKS) the other indirectly by promoting blood destruction.

II. The excretion of bilirubin with the bile was studied in the following way:

A rat was anaesthetized with urethane, and the duodenum was ligated by the pylorus after washing with about 5 cc. of saline. The duodenum was emptied by gentle digital pressure and a second ligature was made about 7 cm more distally. In this way the bile-duct opens into the ligated intestinal loop. The peritoneum and the skin were closed, and the animal was put away for one hour. Consecutively the ligated loop was taken from the animal and the contents brought into a small centrifuging glass. After centrifuging, the clear yellow liquid was sucked up in a syringe, the amount was measured and the whole diluted with water to 10 cc.

The yellow colour was compared with the colour of diluted bile of a normal rat arbitrarily chosen as a standard (100). The yellow colour must originate from bilirubin only, as it was impossible to bring the yellow substance in chloroform, unless the mixture was acidified with strong hydrochloric acid, after which the yellow substance passed *quantitatively* into the chloroform. We used this rather primitive test method for the estimation of bilirubin because the usual methods failed with rats' bile (HYMANS VAN DEN BERGH test, method of SABATINI). The latter method which was designed for the estimation of bilirubin in urine was also tried after addition of diluted urine to the bile. Some colour was obtained, but for our purpose the method was not satisfactory in a quantitative sense, so that we sticked to the primitive method described above.

The use of a rat as a "standard rat" made it impossible to compare the results directly. Therefore we calculated the percentage of difference between each hypophysectomized rat and each normal rat in every group, in this way eliminating the influence of the accidental choice of the "standard rat". These percentages of difference could be averaged for the different groups.

In a first experimental series, hypophysectomized rats (about 10 days after the operation) were compared with normal rats. Table II shows that in all operated animals the reticulocyte level was low. In almost all instances the bilirubin excretion was diminished, although the amount of bile was not significantly different. The average percentage of decrease, calculated as mentioned before, amounted to 31.7 %.

In a second series hypophysectomized rats treated with crude pituitary extract were compared with normal rats. Table III proves that the number of reticulocytes was normal or above normal and that the excretion of

bilirubin was certainly not less than in the normal rats. There even was an average increase of 18.5 % compared with normal.

Thus it could be shown that hypophysectomized rats excrete less bilirubin than normal rats, and that this failure can be corrected by the injection of crude pituitary extract.

These results are in complete accordance with the theory that blood-demolition is diminished in the absence of the hypophysis.

TABLE II.

Bilirubin excretion in normal and hypophysectomizid rats.

Number of rat	Body weight on day of experiment	Retic. $^0/_{00}$ on day of experiment	cc of bile	Total bilirub.	Days after operation	Remarks
B 7153	95	0	0.35	60	9	Hyp. ect.
B 7154	126	4	0.60	73	9	„ „
B 7155	111	36	0.40	105	—	Normal
B 7156	93	68	0.45	100	—	„
B 7157	120	2	0.40	35	10	Hyp. ect.
B 7158	116	4	0.60	64	10	„ „
B 7159	142	36	0.36	94	—	Normal
B 7160	110	20	0.40	100	—	„
B 7161	123	0	0.25	33.5	10	Hyp. ect.
B 7162	99	2	0.22	36.5	10	„ „
B 7163	126	32	0.25	100	—	Normal
B 7164	95	72	0.22	46	—	„
B 7165	112	0	0.25	70	13	Hyp. ect.
B 7166	132	0	0.22	65	13	„ „
B 7242	112	2	0.25	66	13	„ „
B 7243	115	44	0.33	61	—	Normal
B 7244	133	60	0.52	100	—	„
B 7245	134	0	0.56	86	13	Hyp. ect.
B 7246	100	0	0.36	61	13	„ „
B 7247	94	48	0.35	104	—	Normal
B 7248	96	72	0.28	100	—	„

TABLE III.

Bilirubin excretion in normal and hypophysectomized rats treated with pituitary extract

Number of rat	Body weight on day of experiment	Retic. $^0/_{00}$ on day of experiment	cc of bile	Total bilirub.	Days after operation	Remarks
B 7102	186	108	0.60	185	15	Hyp. ect.
B 7103	176	100	0.35	115	15	,, ,,
—	140	44	0.50	100	—	Normal
—	180	100	0.40	115	—	,,
B 7223	131	44	0.33	108	9	Hyp. ect.
B 7224	114	56	0.39	119	9	,, ,,
B 7225	135	64	0.42	117	9	,, ,,
B 7261	146	76	0.30	100	—	Normal
B 7262	114	52	0.16	86	—	,,
B 7263	106	120	0.20	79	—	,,
B 7226	132	72	0.60	64	11	Hyp. ect.
B 7227	131	136	0.40	66	11	,, ,,
B 7228	142	92	0.58	76	11	,, ,,
B 7264	154	60	0.31	100	—	Normal
B 7265	163	68	0.31	86	—	,,
B 7266	150	32	0.35	49	—	,,

The hypophysectomized rats were daily subcutaneously injected with 2 × 0.5 cc of crude alkaline anterior lobe extract.

Summary:

1. Crude alkaline pituitary extracts can be separated in two fractions, one promoting blood destruction and causing a hemolytic anemia, the other causing sometimes a slight polycythemia.

2. Hypophysectomized rats excrete less bilirubin with the bile than normal rats. After the injection of pituitary extract normal quantities are excreted.

These facts are *direct* arguments for the soundness of our theory that the hypophysis contains some factor promoting erythrocyte destruction.

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Medicine. — *Kompendium aus „Seh-Hören".* Von LOTHAR GOLDSCHMIDT.
(Communicated by Prof. A. DE KLEYN.)

(Communicated at the meeting of June 29, 1940.)

Durch einen vor ca. 15 Jahren erlittenen Eisenbahnunfall verlor ich — zum Teil — mein früheres normales Hörvermögen und wurde dadurch veranlasst, mich eingehender mit der Gebärdensprache zu befassen. In diesem Zusammenhang will ich nur darauf hinweisen, dass bereits bei den Völkern des Altertums Zeichensprachen bestanden, und dass das Interesse für die Zeichen- und Gebärdensprache sich bis zum heutigen Tage nicht verminderte. (Siehe W. WUNDT's Völkerpsychologie, G. MALLERY's Sign Language among North American Indians, neuerdings M. CRITCHLEY: „The Language of Gesture", die zur Zeit angewandten Versuche durch Sir R. PAGET, Royal Academy of Great Britain, usf.)

Meine Arbeiten, die sich auf ein grosses empirisches Material gründen, werden durch die bekannte Fingersprache der Taubstummen nicht ersetzt. Bei meinen Methoden habe ich besonders die H ö r e n d e n berücksichtigt, die sich b e i d e r B e r u f s a r b e i t unter ungünstigen Umständen mit der Lautsprache s c h w e r verständigen können, während mit den von mir beschriebenen Systemen die Schwierigkeiten aufgehoben werden können.

a. „G e s t o" = Gebärdenstenographie.
(Auf hieroglyphischer Grundlage.)
Einhand ≐ Sprache für alle Sprachen.

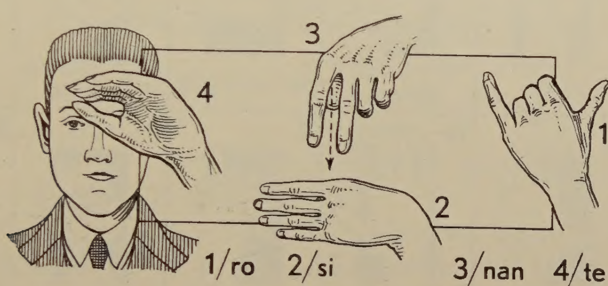
b. „L a n g e s t" = Weltgebärdensprache.
(Möglichkeit einer relativ leicht fassbaren internationalen Verständigung mit einfachster Grammatik und Wörterbuch.)

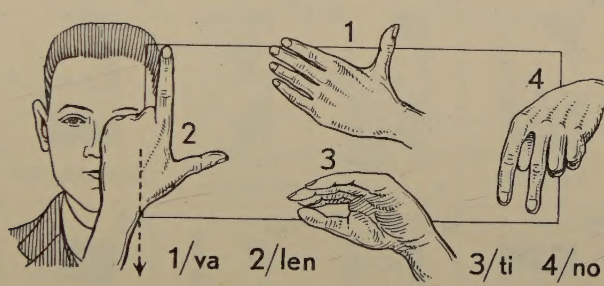
c. „I n s i g" = Internationale Handsignale.
(Analog den Flaggensignalen der Marine.)

„Gebärdenstenographie". „G e s t o" *„Gebaren-Stenografie".*
„Gesture-Shorthand". *„Sténographie des gestes".*

Die Ausführung: erfolgt mit e i n e r Hand, der Einheitlichkeit halber mit der linken, und damit evtl. die rechte gleichzeitig zum Schreiben, Sortieren, ect. benutzt werden kann. Der Zeigeflächenumfang ist aus Schema ersichtlich. A u t o m a t i s c h folgt dem jeweilig an einer der 4 Seiten gezeigten Handbuchstaben der Vokal. Also: oben = *a*, links

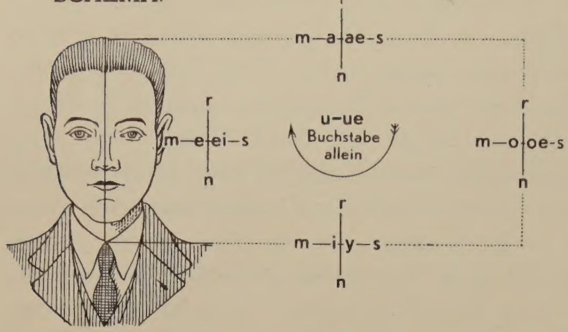
seitwärts = o, unten = *i*, in Nasennähe = e. Wird der Handbuchstabe beim Zeigen dabei etwas gedreht, ist es: *ae, oe, ei,* und unten: y. U wird falls nicht allein, durch Halbkreisbewegung in der Mitte der Zeigefläche gezeigt. Ebenfalls dort die allein stehenden Buchstaben. Eine kl. Drehung am Ende der U = Kreisbewegung ergibt: *ue*. Steht Vokal vor dem Konsonanten bezw. am Silbenbeginn oder Ende, genügt Hinweis nach der betreffenden Seite. Folgt dem Vokal ein *n*, macht man eine kurze Bewegung nach unten, bei *r* = nach oben, *s* = zur Seite, bei *m* = zum Gesicht. (s. Schema). Wiederholung des vorhergehenden Buchstabens ergibt sich aus kurzer Hin- und Zurückbewegung. Wie bei der Stenographie können nicht hörbare sowie Doppel-Laute und -Konsonanten nur einmal gezeigt werden. (= phonetische Wiedergabe). Weitere Kürzungen ergibt die Praxis und evtl. Kombination mit der Gebärdensprache. („Langest".)





Beispiele: „garage" = *g, r,* – oben, *g* – in Nasennähe. — „malaga" = *m, l, g,* – oben. — „faust" = *f* von oben in U-Halbkreis der in *st* endigt. „se(he)n, ste(he)n, we(he)n, ge(he)n" usw. = den betreffenden Buchstaben von Nasennähe abwärts (etwa Kinn) zeigen. — „schein, mein, dein, rein, sein, fein, wein, bein" usw. = Buchstabe in Nasennähe mit Drehung und Abwärtsbewegung. (wie oben.) — „valentino" (s. Skizze) = *v* – oben, *l* – Nasennähe mit Abwärtsbewegung, *t* – unten, *n* – seitwärts. — „rosinante" (s. Skizze) *r* – seitwärts, *s* – unten, *n* – oben mit Abwärtsbewegung, *t* – Nasennähe.

SCHEMA:



Vokalfolge-Grundstellung:
ist Linienschnittpunkt auf Schema.

Stopzeichen:
Kl. Luftquerstrich an jedem Wortende.

Abkürzungen für das Handbuchstabenalphabet:
H = Handfläche, Z = Zeigefinger, D = Daumen, F = Finger, K = Kleine Finger, M = Mittelfinger.

„Weltgebärdensprache" „Language of gestures of the world"

„L a n g e s t".

„Wereld-gebarentaal" „Langue des gestes de la monde"

Die durch den Film, Sport, Foto, Reklame usw. weltverbreitete natürliche Gebärde ist heute Gemeingut aller Menschen und formt sich wie die Lautsprache nach der jeweiligen Umgebung ständig neu. Man versetze sich in die Lage eines Menschen, der in ein fremdes Land kommt dessen Sprache er nicht versteht. — Für die Satzbildung wird hier erstmalig eine einfache Grammatik gebildet. ebenfalls als Hinweis ein Gebärdenwörterbuch.

Gebärdengrammatik: (Auszug)

Artikel:	Fürwort:	
	ich, mir, mich, =	auf sich zeigen
	du, dir,	auf Gegenüber zeigen
der	er (Mann)	Seitenstrich, Handfläche nach innen
die	sie (Frau)	Halbkreis
das, ein		Handfläche nach unten
	wir,	Kreis
	Jhr	2 × vorwärts zeigen
die (Mehrzahl)	sie	2 Seitenstriche
man		Hand auf Schulter

Besitzanzeigende Fürworte:	Desgleichen + entsprechender Hinweis m. Daumen.
haben (mit Abarten)	Hand auf Brust
sein (mit Abarten)	Hand von Brust abwärts (= kl. Körperstrich)
Vergangenheit	Hand rückwärts über Schulter
Zukunft	Hand vorwärts
Frageform	Hand jeweils drehen (? = Daumen auf Handfläche)
Befehlsform	Faust vorwärts
Ohne Zeitangabe	gilt Gegenwart, mit Artikel: Substantiv.

Beispiele: „ich habe gehabt" = auf sich + rückwärts + auf Brust zeigen. „sie wird sein" = Halbkreis + vorwärts + Körperstrich. „sie würden haben" = 2 Seitenstriche + Vorwärtsdrehung + Brust. „wir haben" = Kreis + Brust. — „wir sind" = Kreis + Körperstrich. — „würden wir sein?" = Hand vorw. drehen + Kreis + Körperstrich + ? (s. oben) „sie hatte kein Geld" = Halbkreis + rückw. + Hand schütteln + Geldhand (= Daumen u. Zeigef. reiben).

„Internationale Handsignale" „I n s i g". „Internationale Handseinen"
„International Signs of the hand" „Signales de la main internationales".

Die hierfür angelegte Sammlung scheint notwendig zu sein. Denn durch die Gleichart der Maschinen, Arbeitsmethoden, Sport usw. sind sie auf allen Gebieten gleichartig, in denen die Lautspracheverständigung durch Lärm, Entfernung usw. erschwert oder wie beim Radiosender, Tonfilmstudio, Verkehrsregelung usw. unmöglich ist. Ferner erfordern die vielen durch Lärm, Staub, Ausdünstung, (Vergiftungsgefahr, Berufsschwerhörigkeit, Berufskrankheiten des Ohres und der oberen Luftwege usw.) gesundheitsgefährdende Arbeitsstätten, Abdichtung der betreffenden Organe, was wie psychologisch verständlich, ohne Zeichenverständigungsmöglichkeit immer unterbleiben wird. „I n s i g" wird auf Gebieten angewandt deren grosser Umfang garnicht genug bekannt ist. Sie sind international notwendig und sollten in den Gewerbe- und Handelsschulen gelehrt werden. Die „Technische-Hochschule, Charlottenburg" und der „Verein Deutscher Ingenieure", Berlin, haben erst jetzt wieder das Bestehen einer derartigen Sammlung verneint. Das Interesse der Chefingenieure der Werften, Webereien, Fabriken Krananlagen, Rotations-Schiffs-Maschinenräume (Pfadfinder-Luftschutz-Gasmaskenträger) usw. bestätigt die Zweckmässigkeit einer derartigen Sammlung.

Vorliegende bereits erprobte 3 Systeme (evtl. auch kombiniert angewandt) sind ein neuer Beitrag der Psychotechnik im Dienst der Betriebswirtschaft ausser der sonstigen universalen Bedeutung. Ein Erlernen dieser — neuen körperlich = geistigen Eigenschaft — ist kaum nötig. Die Absicht des Verfassers ist, nicht nur Schwerhörigen eine leichte (auch evtl. „ablesbare") Verständigung ohne Apparate — die, gleichviel welcher Konstruktion, nicht wie es beim Auge bei herabgesetztem Sehvermögen das jeweilige Glas tut, das Hörfähigkeitsminimum ausgleichen können — zu bieten sondern auch die bisher noch nicht vorhandenen aber notwendigen international verständlichen für *Normalhörende geeignete Gebärdensysteme* gefunden zu haben.